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Review Article

Electromagnetic Properties of Neutrinos

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In this paper, we discuss the main theoretical aspects and experimental effects of neutrino electromagnetic properties. We start with a general description of the electromagnetic form factors of Dirac and Majorana neutrinos. Then, we discuss the theory and phenomenology of the magnetic and electric dipole moments, summarizing the experimental results and the theoretical predictions. We discuss also the phenomenology of a neutrino charge radius and radiative decay. Finally, we describe the theory of neutrino spin and spin-flavor precession in a transverse magnetic field and we summarize its phenomenological applications.

1. Introduction

The investigation of neutrino properties is one of the most active fields of research in current high-energy physics. Neutrinos are special particles, because they interact very weakly and their masses are much smaller than those of the other fundamental fermions (charged leptons and quarks). In the Standard Model neutrinos are massless and have only weak interactions. However, the observation of neutrino oscillations by many experiments (see [1–4]) imply that neutrinos are massive and mixed. Therefore, the Standard Model must be extended to account for neutrino masses. In many extensions of the Standard Model neutrinos acquire also electromagnetic properties through quantum loops effects. Hence, the theoretical and experimental study of neutrino electromagnetic interactions is a powerful tool in the search for the fundamental theory beyond the Standard Model. Moreover, the electromagnetic interactions of neutrinos can generate important effects, especially in astrophysical environments, where neutrinos propagate for long distances in magnetic fields both in vacuum and in matter.

In this paper, we review the theory and phenomenology of neutrino electromagnetic interactions. After a derivation of all the possible types of electromagnetic interactions of Dirac and Majorana neutrinos, we discuss their effects in terrestrial and astrophysical environments and the corresponding experimental results. In spite of many efforts in the search of neutrino electromagnetic interactions, up to now there is no positive experimental indication in favor of their existence. However, the existence of neutrino masses and mixing implies that nontrivial neutrino electromagnetic properties are plausible and experimentalists and theorists are eagerly looking for them.

In this review, we use the notation and conventions in [1]. When we consider neutrino mixing, we have the relation:

$$\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k} \nu_{kL} \quad (\alpha = e, \mu, \tau) \tag{1.1}$$

between the left-handed components of the three flavor neutrino fields ν_e, ν_μ, ν_τ and the left-handed components of three massive neutrino fields ν_k with masses m_k ($k = 1, 2, 3$). The 3×3 mixing matrix U is unitary ($U^\dagger = U^{-1}$).

Neutrino electromagnetic properties are discussed in [4–7], and in the previous reviews in [8–14].

The structure of this paper is as follows. In Section 2, we discuss the general form of the electromagnetic interaction of Dirac and Majorana neutrinos, which is expressed in terms of form factors, and the derivation of the form factors in gauge models. In Section 3, we discuss the phenomenology of the neutrino magnetic and electric dipole moments in laboratory experiments. These are the most studied electromagnetic properties of neutrinos, both experimentally and theoretically. In Section 4, we review the theory and experimental constraints on the neutrino charge radius. In Section 5, we discuss neutrino radiative decay and the astrophysical bounds on a neutrino magnetic moment obtained from the study of plasmon decay in stars. In Section 6 we discuss neutrino spin and spin-flavor precession. In conclusion, in Section 7, we summarize the status of our knowledge of neutrino electromagnetic properties and we discuss the prospects for future research.

2. Electromagnetic Form Factors

The importance of neutrino electromagnetic properties was first mentioned by Pauli in 1930, when he postulated the existence of this particle and discussed the possibility that the neutrino might have a magnetic moment. Systematic theoretical studies of neutrino electromagnetic properties started after it was shown that in the extended Standard Model with right-handed neutrinos the magnetic moment of a massive neutrino is, in general, nonvanishing and that its value is determined by the neutrino mass [15–23].

Neutrino electromagnetic properties are important because they are directly connected to fundamentals of particle physics. For example, neutrino electromagnetic properties can be used to distinguish Dirac and Majorana neutrinos (see [21, 22, 24–27]) and also as probes of new physics that might exist beyond the Standard Model (see [28–30]).

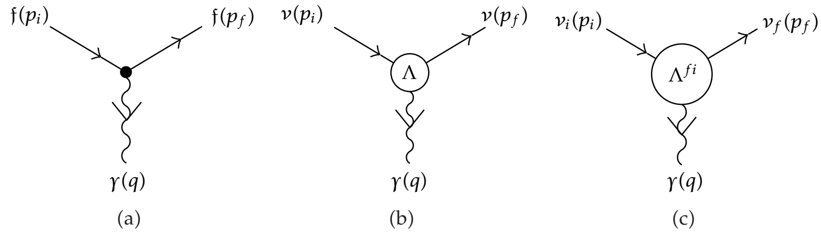

Figure 1: Tree-level coupling of a charged fermion $\mathfrak{f}$ with a photon γ (a), effective coupling of a neutrino ν with a photon (b), and effective coupling of neutrinos with a photon taking into account possible transitions between two different initial and final massive neutrinos ν_i and ν_f (c).

2.1. Dirac Neutrinos

In the Standard Model, the interaction of a fermionic field $\mathfrak{f}(x)$ with the electromagnetic field $A^\mu(x)$ is given by the interaction Hamiltonian:

$$\mathcal{H}_{\text{em}}(x) = j_\mu(x)A^\mu(x) = q_{\mathfrak{f}}\bar{\mathfrak{f}}(x)\gamma_\mu\mathfrak{f}(x)A^\mu(x), \tag{2.1}$$

where $q_{\mathfrak{f}}$ is the charge of the fermion $\mathfrak{f}$. Figure 1(a) shows the corresponding tree-level Feynman diagram (the photon γ is the quantum of the electromagnetic field $A^\mu(x)$).

For neutrinos, the electric charge is zero and there are no electromagnetic interactions at tree-level (however, in some theories beyond the Standard Model neutrinos can be millicharged particles (see [13])). However, such interactions can arise at the quantum level from loop diagrams at higher order of the perturbative expansion of the interaction. In the one-photon approximation, the electromagnetic interactions of a neutrino field $\nu(x)$ can be described by the effective interaction Hamiltonian:

$$\mathcal{H}_{\text{eff}}(x) = j_\mu^{\text{eff}}(x)A^\mu(x) = \overline{\nu}(x)\Lambda_\mu\nu(x)A^\mu(x), \tag{2.2}$$

where $j_\mu^{\text{eff}}(x)$ is the effective neutrino electromagnetic current four-vector and Λ_μ is a 4×4 matrix in spinor space which can contain space-time derivatives, such that $j_\mu^{\text{eff}}(x)$ transforms as a four-vector. Since radiative corrections are generated by weak interactions which are not invariant under a parity transformation, $j_\mu^{\text{eff}}(x)$ can be a sum of polar and axial parts. The corresponding diagram for the interaction of a neutrino with a photon is shown in Figure 1(b), where the blob represents the quantum loop contributions.

We are interested in the neutrino part of the amplitude corresponding to the diagram in Figure 1(b), which is given by the matrix element:

$$\left\langle \nu(p_f, h_f) \middle| j_\mu^{\text{eff}}(x) \middle| \nu(p_i, h_i) \right\rangle, \tag{2.3}$$

where $p_i(p_f)$ and $h_i(h_f)$ are the four-momentum and helicity of the initial (final) neutrino. Taking into account that

$$\partial^\mu j_\mu^{\text{eff}}(x) = i\left[\mathcal{P}^\mu, j_\mu^{\text{eff}}(x)\right], \tag{2.4}$$

where $\mathcal{P}^\mu$ is the four-momentum operator which generate translations, the effective current can be written as

$$j_\mu^{\text{eff}}(x) = e^{i\mathcal{P}\cdot x} j_\mu^{\text{eff}}(0) e^{-i\mathcal{P}\cdot x}. \tag{2.5}$$

Since $\mathcal{P}^\mu|\nu(p)\rangle = p^\mu|\nu(p)\rangle$, we have

$$\left\langle \nu(p_f) \left| j_\mu^{\text{eff}}(x) \right| \nu(p_i) \right\rangle = e^{i(p_f - p_i)\cdot x} \left\langle \nu(p_f) \left| j_\mu^{\text{eff}}(0) \right| \nu(p_i) \right\rangle, \tag{2.6}$$

where we suppressed for simplicity the helicity labels which are not of immediate relevance. Here we see that the unknown quantity which determines the neutrino-photon interaction is $\langle \nu(p_f)|j_\mu^{\text{eff}}(0)|\nu(p_i)\rangle$. Considering that the incoming and outgoing neutrinos are free particles which are described by free Dirac fields with the standard Fourier expansion in (2.139) of [1], we have

$$\left\langle \nu(p_f) \left| j_\mu^{\text{eff}}(0) \right| \nu(p_i) \right\rangle = \overline{u}(p_f) \Lambda_\mu(p_f, p_i) u(p_i). \tag{2.7}$$

The electromagnetic properties of neutrinos are embodied by $\Lambda_\mu(p_f, p_i)$, which is a matrix in spinor space and can be decomposed in terms of linearly independent products of Dirac γ matrices and the available kinematical four-vectors p_i and p_f. The most general decomposition can be written as (see [11])

$$\begin{aligned}\Lambda_\mu(p_f, p_i) = {} & f_1\left(q^2\right) q_\mu + f_2\left(q^2\right) q_\mu \gamma_5 + f_3\left(q^2\right) \gamma_\mu + f_4\left(q^2\right) \gamma_\mu \gamma_5 \\ & + f_5\left(q^2\right) \sigma_{\mu\nu} q^\nu + f_6\left(q^2\right) \epsilon_{\mu\nu\rho\gamma} q^\nu \sigma^{\rho\gamma},\end{aligned} \tag{2.8}$$

where $f_k(q^2)$ are six Lorentz-invariant form factors ($k = 1, \ldots, 6$) and q is the four-momentum of the photon, which is given by

$$q = p_i - p_f, \tag{2.9}$$

from energy-momentum conservation. Notice that the form factors depend only on q^2, which is the only available Lorentz-invariant kinematical quantity, since $(p_i + p_f)^2 = 4m^2 - q^2$. Therefore, $\Lambda_\mu(p_f, p_i)$ depends only on q and from now on we will denote it as $\Lambda_\mu(q)$.

Since the Hamiltonian and the electromagnetic field are Hermitian ($\mathcal{H}_{\text{eff}}^\dagger = \mathcal{H}_{\text{eff}}$ and $A^{\mu\dagger} = A^\mu$), the effective current must be Hermitian, $j_\mu^{\text{eff}\dagger} = j_\mu^{\text{eff}}$. Hence, we have

$$\left\langle \nu(p_f) \left| j_\mu^{\text{eff}}(0) \right| \nu(p_i) \right\rangle = \left\langle \nu(p_i) \left| j_\mu^{\text{eff}}(0) \right| \nu(p_f) \right\rangle^*, \tag{2.10}$$

which leads to

$$\Lambda_\mu(q) = \gamma^0 \Lambda_\mu^\dagger(-q) \gamma^0. \tag{2.11}$$

This constraint implies that

$$f_2, f_3, f_4 \quad \text{are real,} \tag{2.12}$$

$$f_1, f_5, f_6 \quad \text{are imaginary.} \tag{2.13}$$

The number of independent form factors can be reduced by imposing current conservation, $\partial^\mu j_\mu^{\text{eff}}(x) = 0$, which is required by gauge invariance (i.e., invariance of $\mathscr{H}_{\text{eff}}(x)$ under the transformation $A^\mu(x) \to A^\mu(x) + \partial^\mu \varphi(x)$ for any $\varphi(x)$, which leaves invariant the electromagnetic tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$). Using (2.4), current conservation implies that

$$\left\langle \nu(p_f) \right| \left[\mathcal{P}^\mu, j_\mu^{\text{eff}}(0) \right] \left| \nu(p_i) \right\rangle = 0. \tag{2.14}$$

Hence, in momentum space, we have the constraint:

$$q^\mu \, \overline{u}(p_f) \Lambda_\mu(q) u(p_i) = 0, \tag{2.15}$$

which implies that

$$f_1\left(q^2\right) q^2 + f_2\left(q^2\right) q^2 \gamma_5 + 2m f_4\left(q^2\right) \gamma_5 = 0. \tag{2.16}$$

Since γ_5 and the unity matrix are linearly independent, we obtain the constraints:

$$f_1\left(q^2\right) = 0, \quad f_4\left(q^2\right) = -\frac{f_2(q^2) q^2}{2m}. \tag{2.17}$$

Therefore, in the most general case, consistent with Lorentz and electromagnetic gauge invariance, the vertex function is defined in terms of four form factors [25–27]:

$$\Lambda_\mu(q) = f_Q\left(q^2\right)\gamma_\mu - f_M\left(q^2\right) i\sigma_{\mu\nu} q^\nu + f_E\left(q^2\right)\sigma_{\mu\nu} q^\nu \gamma_5 + f_A\left(q^2\right)\left(q^2\gamma_\mu - q_\mu \not{q}\right)\gamma_5. \tag{2.18}$$

where $f_Q = f_3$, $f_M = i f_5$, $f_E = -2i f_6$, and $f_A = -f_2/2m$ are the real charge, dipole magnetic and electric, and anapole neutrino form factors. For the coupling with a real photon ($q^2 = 0$),

$$f_Q(0) = q, \qquad f_M(0) = \mu, \qquad f_E(0) = \epsilon, \qquad f_A(0) = a, \tag{2.19}$$

where q, μ, ϵ, and a are, respectively, the neutrino charge, magnetic moment, electric moment and anapole moment. Although above we stated that $q = 0$, here we did not enforce this equality because in some theories beyond the Standard Model neutrinos can be millicharged particles (see [13]).

Now, it is interesting to study the properties of $\mathscr{H}_{\text{eff}}(x)$ under a CP transformation, in order to find which of the terms in (2.18) violate CP. Let us consider the active CP transformation:

$$\text{U}_{\text{CP}} \nu(x) \text{U}_{\text{CP}}^\dagger = \xi^{\text{CP}} \gamma^0 \mathcal{C} \overline{\nu}^T(x_{\text{P}}), \tag{2.20}$$

where ξ^{CP} is a phase, $\mathcal{C}$ is the charge-conjugation matrix (such that $\mathcal{C}\gamma_\mu^T\mathcal{C}^{-1} = -\gamma_\mu$, $\mathcal{C}^\dagger = \mathcal{C}^{-1}$ and $\mathcal{C}^T = -\mathcal{C}$), and $x_{\mathrm{P}}^\mu = x_\mu$. For the Standard Model electric current $j_\mu(x)$ in (2.1), we have

$$j_\mu(x) \xrightarrow{\mathrm{CP}} \mathrm{U}_{\mathrm{CP}} j_\mu(x) \mathrm{U}_{\mathrm{CP}}^\dagger = -j^\mu(x_{\mathrm{P}}). \tag{2.21}$$

Hence, the Standard Model electromagnetic interaction Hamiltonian $\mathcal{H}_{\mathrm{em}}(x)$ is left invariant by (the transformation $x \to x_{\mathrm{P}}$ is irrelevant since all amplitudes are obtained by integrating over d^4x)

$$A_\mu(x) \xrightarrow{\mathrm{CP}} -A^\mu(x_{\mathrm{P}}). \tag{2.22}$$

CP is conserved in neutrino electromagnetic interactions (in the one-photon approximation) if $j_\mu^{\mathrm{eff}}(x)$ transforms as $j_\mu(x)$:

$$\mathrm{CP} \Longleftrightarrow \mathrm{U}_{\mathrm{CP}} j_\mu^{\mathrm{eff}}(x) \mathrm{U}_{\mathrm{CP}}^\dagger = -j_{\mathrm{eff}}^\mu(x_{\mathrm{P}}). \tag{2.23}$$

For the matrix element (2.7), we obtain

$$\mathrm{CP} \Longleftrightarrow \Lambda_\mu(q) \xrightarrow{\mathrm{CP}} -\Lambda^\mu(q). \tag{2.24}$$

One can find that under a CP transformation we have

$$\Lambda_\mu(q) \xrightarrow{\mathrm{CP}} \gamma^0 \mathcal{C} \Lambda_\mu^T(q_{\mathrm{P}}) \mathcal{C}^\dagger \gamma^0, \tag{2.25}$$

with $q_{\mathrm{P}}^\mu = q_\mu$. Using the form factor expansion in (2.18), we obtain

$$\Lambda_\mu(q) \xrightarrow{\mathrm{CP}} -\left[f_Q\left(q^2\right)\gamma^\mu - f_M\left(q^2\right) i\sigma^{\mu\nu} q_\nu - f_E\left(q^2\right)\sigma^{\mu\nu} q_\nu \gamma_5 + f_A\left(q^2\right)\left(q^2\gamma^\mu - q^\mu \not{q}\right)\gamma_5\right]. \tag{2.26}$$

Therefore, only the electric dipole form factor violates CP:

$$\mathrm{CP} \Longleftrightarrow f_E\left(q^2\right) = 0. \tag{2.27}$$

So far, we have considered only one massive neutrino field $\nu(x)$, but according to the mixing relation (1.1), the three flavor neutrino fields ν_e, ν_μ, ν_τ are unitary linear combinations of three massive neutrinos ν_k $(k = 1, 2, 3)$. Therefore, we must generalize the discussion to the case of more than one massive neutrino field. The effective electromagnetic interaction Hamiltonian in (2.2) is generalized to

$$\mathcal{H}_{\mathrm{eff}}(x) = j_\mu^{\mathrm{eff}}(x) A^\mu(x) = \sum_{k,j=1}^{3} \overline{\nu_k}(x) \Lambda_\mu^{kj} \nu_j(x) A^\mu(x), \tag{2.28}$$

where we take into account possible transitions between different massive neutrinos. The physical effect of $\mathscr{H}_{\text{eff}}$ is described by the effective electromagnetic vertex in Figure 1(c), with the neutrino matrix element:

$$\left\langle \nu_f(p_f) \middle| j_\mu^{\text{eff}}(0) \middle| \nu_i(p_i) \right\rangle = \overline{u_f}(p_f)\Lambda_\mu^{fi}(p_f, p_i)\mathrm{u}_i(p_i). \tag{2.29}$$

As in the case of one massive neutrino field, $\Lambda_\mu^{fi}(p_f, p_i)$ depends only on the four-momentum q transferred to the photon and can be expressed in terms of six Lorentz-invariant form factors:

$$\begin{aligned} \Lambda_\mu^{fi}(q) = {} & f_1^{fi}\left(q^2\right)q_\mu + f_2^{fi}\left(q^2\right)q_\mu\gamma_5 + f_3^{fi}\left(q^2\right)\gamma_\mu + f_4^{fi}\left(q^2\right)\gamma_\mu\gamma_5 \\ & + f_5^{fi}\left(q^2\right)\sigma_{\mu\nu}q^\nu + f_6^{fi}\left(q^2\right)\epsilon_{\mu\nu\rho\gamma}q^\nu\sigma^{\rho\gamma}. \end{aligned} \tag{2.30}$$

The Hermitian nature of j_μ^{eff} implies that $\langle \nu_f(p_f)|j_\mu^{\text{eff}}(0)|\nu_i(p_i)\rangle = \langle \nu_i(p_i)|j_\mu^{\text{eff}}(0)|\nu_f(p_f)\rangle^*$, leading to the constraint:

$$\Lambda_\mu^{fi}(q) = \gamma^0\left[\Lambda_\mu^{if}(-q)\right]^\dagger\gamma^0. \tag{2.31}$$

Considering the 3×3 form factor matrices f_k in the space of massive neutrinos with components f_k^{fi} for $k = 1, \ldots, 6$, we find that

$$f_2, f_3, f_4 \quad \text{are Hermitian}, \tag{2.32}$$

$$f_1, f_5, f_6 \quad \text{are anti-Hermitian}. \tag{2.33}$$

Following the same method used in (2.4)–(2.16), one can find that current conservation implies the constraints:

$$f_1^{fi}\left(q^2\right)q^2 + f_3^{fi}\left(q^2\right)(m_f - m_i) = 0, \qquad f_2^{fi}\left(q^2\right)q^2 + f_4^{fi}\left(q^2\right)(m_f + m_i) = 0. \tag{2.34}$$

Therefore, we obtain

$$\Lambda_\mu^{fi}(q) = \left(\gamma_\mu - q_\mu\frac{\not{q}}{q^2}\right)\left[f_Q^{fi}\left(q^2\right) + f_A^{fi}\left(q^2\right)q^2\gamma_5\right] - i\sigma_{\mu\nu}q^\nu\left[f_M^{fi}\left(q^2\right) + if_E^{fi}\left(q^2\right)\gamma_5\right], \tag{2.35}$$

where $f_Q^{fi} = f_3^{fi}$, $f_M^{fi} = if_5^{fi}$, $f_E^{fi} = -2if_6^{fi}$ and $f_A^{fi} = -f_2^{fi}/(m_f + m_i)$, with

$$f_\Omega^{fi} = \left(f_\Omega^{if}\right)^* \quad (\Omega = Q, M, E, A). \tag{2.36}$$

Note that since $\overline{u_f}(p_f)\,\not{q}u_i(p_i) = (m_f - m_i)\,\overline{u_f}(p_f)u_i(p_i)$, if $f = i$, (2.35) correctly reduces to (2.18).

The form factors with $f = i$ are called "diagonal," whereas those with $f \neq i$ are called "off-diagonal" or "transition form factors." This terminology follows from the expression:

$$\Lambda_\mu(q) = \left(\gamma_\mu - q_\mu \frac{\not{q}}{q^2}\right)\left[f_Q\left(q^2\right) + f_A\left(q^2\right)q^2\gamma_5\right] - i\sigma_{\mu\nu}q^\nu\left[f_M\left(q^2\right) + if_E\left(q^2\right)\gamma_5\right], \tag{2.37}$$

in which $\Lambda_\mu(q)$ is a 3×3 matrix in the space of massive neutrinos expressed in terms of the four Hermitian 3×3 matrices of form factors:

$$f_\Omega = f_\Omega^\dagger \quad (\Omega = Q, M, E, A). \tag{2.38}$$

For the coupling with a real photon ($q^2 = 0$), we have

$$f_Q^{fi}(0) = q_{fi}, \qquad f_M^{fi}(0) = \mu_{fi}, \qquad f_E^{fi}(0) = \epsilon_{fi}, \qquad f_A^{fi}(0) = a_{fi}, \tag{2.39}$$

where q_{fi}, μ_{fi}, ϵ_{fi}, and a_{fi} are, respectively, the neutrino charge, magnetic moment, electric moment, and anapole moment of diagonal ($f = i$) and transition ($f \neq i$) types.

Considering now CP invariance, the transformation (2.23) of $j_\mu^{\text{eff}}(x)$ implies the constraint in (2.24) for the $N \times N$ matrix $\Lambda_\mu(q)$ in the space of massive neutrinos. Using (2.20), we obtain

$$\Lambda_\mu^{fi}(q) \xrightarrow{\text{CP}} \xi_f^{\text{CP}}\xi_i^{\text{CP}*}\gamma^0\mathcal{C}\left[\Lambda_\mu^{if}(q_{\text{P}})\right]^T\mathcal{C}^\dagger\gamma^0, \tag{2.40}$$

where ξ_k^{CP} is the CP phase of ν_k. Since the three massive neutrinos take part to standard charged-current weak interactions, their CP phases are equal if CP is conserved (see [1]). Hence, we have

$$\Lambda_\mu^{fi}(q) \xrightarrow{\text{CP}} \gamma^0\mathcal{C}\left[\Lambda_\mu^{if}(q_{\text{P}})\right]^T\mathcal{C}^\dagger\gamma^0. \tag{2.41}$$

Using the form factor expansion in (2.35), we obtain

$$\Lambda_\mu^{fi}(q) \xrightarrow{\text{CP}} -\left\{\left(\gamma^\mu - q^\mu \frac{\not{q}}{q^2}\right)\left[f_Q^{if}\left(q^2\right) + f_A^{if}\left(q^2\right)q^2\gamma_5\right] - i\sigma^{\mu\nu}q_\nu\left[f_M^{if}\left(q^2\right) - if_E^{if}\left(q^2\right)\gamma_5\right]\right\}. \tag{2.42}$$

Imposing the constraint in (2.24), for the form factors, we obtain

$$\text{CP} \Longleftrightarrow \begin{cases} f_\Omega^{fi} = f_\Omega^{if} = \left(f_\Omega^{fi}\right)^* & (\Omega = Q, M, A), \\ f_E^{fi} = -f_E^{if} = -\left(f_E^{fi}\right)^*, & \end{cases} \tag{2.43}$$

where, in the last equalities, we took into account the constraints (2.36). For the Hermitian 3×3 form factor matrices, we obtain that if CP is conserved f_Q, f_M and f_A, are real and symmetric and f_E is imaginary and antisymmetric:

$$\text{CP} \Longleftrightarrow \begin{cases} f_\Omega = f_\Omega^{\text{T}} = f_\Omega^* & (\Omega = Q, M, A), \\ f_E = -f_E^{\text{T}} = -f_E^*. \end{cases} \tag{2.44}$$

Let us now consider antineutrinos. Using for the massive neutrino fields, the Fourier expansion in (2.139) of [1], the effective antineutrino matrix element for $\overline{\nu}_i(p_i) \to \overline{\nu}_f(p_f)$ transitions is given by

$$\left\langle \overline{\nu}_f(p_f) \middle| j_\mu^{\text{eff}}(0) \middle| \overline{\nu}_i(p_i) \right\rangle = -\overline{v_i}(p_i) \Lambda_\mu^{if}(q) v_f(p_f). \tag{2.45}$$

Using the relation:

$$u(p) = \mathcal{C}\overline{v}^T(p), \tag{2.46}$$

we can write it as

$$\left\langle \overline{\nu}_f(p_f) \middle| j_\mu^{\text{eff}}(0) \middle| \overline{\nu}_i(p_i) \right\rangle = \overline{u_f}(p_f) \mathcal{C}\left[\Lambda_\mu^{if}(q)\right]^T \mathcal{C}^\dagger u_i(p_i), \tag{2.47}$$

where transposition operates in spinor space. Therefore, the effective form factor matrix in spinor space for antineutrinos is given by

$$\overline{\Lambda}_\mu^{fi}(q) = \mathcal{C}\left[\Lambda_\mu^{if}(q)\right]^T \mathcal{C}^\dagger. \tag{2.48}$$

Using the properties of the charge-conjugation matrix and the expression (2.35) for $\Lambda_\mu^{if}(q)$, we obtain the antineutrino form factors:

$$\overline{f}_\Omega^{fi} = -f_\Omega^{if} \quad (\Omega = Q, M, E), \tag{2.49}$$

$$\overline{f}_A^{fi} = f_A^{if}. \tag{2.50}$$

Therefore, in particular the diagonal magnetic and electric moments of neutrinos and antineutrinos have the same size with opposite signs, as the charge, if it exists. On the other hand, the diagonal neutrino and antineutrino anapole moments are equal.

2.2. Majorana Neutrinos

A massive Majorana neutrino is a neutral spin 1/2 particle which coincides with its antiparticle. The four degrees of freedom of a massive Dirac field (two helicities and two particle-antiparticle) are reduced to two (two helicities) by the Majorana constraint:

$$\nu_k = \nu_k^c = \mathcal{C}\overline{\nu_k}^T. \tag{2.51}$$

Since a Majorana field has half the degrees of freedom of a Dirac field, it is possible that its electromagnetic properties are reduced. From the relations (2.49) and (2.50) between neutrino and antineutrino form factors in the Dirac case, we can infer that in the Majorana case the charge, magnetic, and electric form factor matrices are antisymmetric and the anapole form factor matrix is symmetric. In order to confirm this deduction, let us calculate the neutrino matrix element corresponding to the effective electromagnetic vertex in Figure 1(c), with the effective interaction Hamiltonian in (2.28), which takes into account possible transitions between two different initial and final massive Majorana neutrinos ν_i and ν_f. Using the neutrino Majorana fields, the Fourier expansion in (6.99) of [1], we obtain

$$\left\langle \nu_f(p_f) \middle| j_\mu^{\text{eff}}(0) \middle| \nu_i(p_i) \right\rangle = \overline{u_f}(p_f)\Lambda_\mu^{fi}(p_f,p_i)u_i(p_i) - \overline{v_i}(p_i)\Lambda_\mu^{if}(p_f,p_i)v_f(p_f). \tag{2.52}$$

Using (2.46), we can write it as

$$\overline{u_f}(p_f)\left\{\Lambda_\mu^{fi}(p_f,p_i) + \mathcal{C}\left[\Lambda_\mu^{if}(p_f,p_i)\right]^T\mathcal{C}^\dagger\right\}u_i(p_i), \tag{2.53}$$

where transposition operates in spinor space. Therefore, the effective form factor matrix in spinor space for Majorana neutrinos is given by

$$\widetilde{\Lambda}_\mu^{fi}(p_f,p_i) = \Lambda_\mu^{fi}(p_f,p_i) + \mathcal{C}\left[\Lambda_\mu^{if}(p_f,p_i)\right]^T\mathcal{C}^\dagger. \tag{2.54}$$

As in the case of Dirac neutrinos, $\Lambda_\mu^{fi}(p_f,p_i)$ depends only on $q = p_f - p_i$ and can be expressed in terms of six Lorentz-invariant form factors according to (2.30). Hence, we can write the 3×3 matrix $\widetilde{\Lambda}_\mu(p_f,p_i)$ in the space of massive Majorana neutrinos as

$$\begin{aligned}\widetilde{\Lambda}_\mu(q) = {}& \widetilde{f}_1\left(q^2\right)q_\mu + \widetilde{f}_2\left(q^2\right)q_\mu\gamma_5 + \widetilde{f}_3\left(q^2\right)\gamma_\mu + \widetilde{f}_4\left(q^2\right)\gamma_\mu\gamma_5 \\ & + \widetilde{f}_5\left(q^2\right)\sigma_{\mu\nu}q^\nu + \widetilde{f}_6\left(q^2\right)\epsilon_{\mu\nu\rho\gamma}q^\nu\sigma^{\rho\gamma},\end{aligned} \tag{2.55}$$

with

$$\begin{aligned}\widetilde{f}_k = f_k + f_k^T \Longrightarrow \widetilde{f}_k = \widetilde{f}_k^T \quad \text{for } k = 1,2,4, \\ \widetilde{f}_k = f_k - f_k^T \Longrightarrow \widetilde{f}_k = -\widetilde{f}_k^T \quad \text{for } k = 3,5,6.\end{aligned} \tag{2.56}$$

Now, we can follow the discussion in Section 2.1 for Dirac neutrinos taking into account the additional constraints (2.56) for Majorana neutrinos. The hermiticity of j_μ^{eff} and current conservation lead to an expression similar to that in (2.37):

$$\widetilde{\Lambda}_\mu(q) = \left(\gamma_\mu - q_\mu \frac{\not{q}}{q^2}\right)\left[\widetilde{f}_Q\left(q^2\right) + \widetilde{f}_A\left(q^2\right)q^2\gamma_5\right] - i\sigma_{\mu\nu}q^\nu\left[\widetilde{f}_M\left(q^2\right) + i\widetilde{f}_E\left(q^2\right)\gamma_5\right], \tag{2.57}$$

with $\widetilde{f}_Q^{fi} = \widetilde{f}_3^{fi}$, $\widetilde{f}_M^{fi} = i\widetilde{f}_5^{fi}$, $\widetilde{f}_E^{fi} = -2i\widetilde{f}_6^{fi}$, and $\widetilde{f}_A^{fi} = -\widetilde{f}_2^{fi}/(m_f + m_i)$. For the Hermitian 3×3 form factor matrices in the space of massive neutrinos,

$$\widetilde{f}_\Omega = \widetilde{f}_\Omega^\dagger \quad (\Omega = Q, M, E, A), \tag{2.58}$$

the Majorana constraints (2.56) imply that

$$\widetilde{f}_\Omega = -\widetilde{f}_\Omega^T \quad (\Omega = Q, M, E), \tag{2.59}$$

$$\widetilde{f}_A = \widetilde{f}_A^T. \tag{2.60}$$

These relations confirm the expectation discussed above that for Majorana neutrinos the charge, magnetic and electric form factor matrices are antisymmetric and the anapole form factor matrix is symmetric.

Since $\widetilde{f}_Q$, $\widetilde{f}_M$, and $\widetilde{f}_E$ are antisymmetric, a Majorana neutrino does not have diagonal charge and dipole magnetic, and electric form factors. It can only have a diagonal anapole form factor. On the other hand, Majorana neutrinos can have as many off-diagonal (transition) form factors as Dirac neutrinos.

Since the form factor matrices are Hermitian as in the Dirac case, $\widetilde{f}_Q$, $\widetilde{f}_M$, and $\widetilde{f}_E$ are imaginary, whereas $\widetilde{f}_A$ is real:

$$\widetilde{f}_\Omega = -\widetilde{f}_\Omega^* \quad (\Omega = Q, M, E), \tag{2.61}$$

$$\widetilde{f}_A = \widetilde{f}_A^*. \tag{2.62}$$

Considering now CP invariance, the case of Majorana neutrinos is rather different from that of Dirac neutrinos, because the CP phases of the massive Majorana fields ν_k are constrained by the CP invariance of the Lagrangian Majorana mass term:

$$\mathcal{L}_M = \frac{1}{2}\sum_k m_k \nu_k^T\, \mathcal{C}^\dagger \nu_k. \tag{2.63}$$

In order to prove this statement, let us first notice that since a massive Majorana neutrino field ν_k is constrained by the Majorana relation in (2.51), only the parity transformation part is effective in a CP transformation:

$$\mathrm{U}_{\mathrm{CP}}\nu_k(x)\mathrm{U}_{\mathrm{CP}}^\dagger = \xi_k^{\mathrm{CP}}\gamma^0\nu_k(x_{\mathrm{P}}). \tag{2.64}$$

Considering the mass term in (2.63), we have

$$U_{\text{CP}} \nu_k^T \, \mathcal{C}^\dagger \nu_k U_{\text{CP}}^\dagger = -\xi_k^{\text{CP}^2} \nu_k^T \mathcal{C}^\dagger \nu_k. \tag{2.65}$$

Therefore,

$$\text{CP} \Longleftrightarrow \xi_k^{\text{CP}} = \eta_k i, \tag{2.66}$$

with $\eta_k = \pm 1$. These CP signs can be different for the different massive neutrinos, even if they all take part to the standard charged-current weak interactions through neutrino mixing, because they can be compensated by the Majorana CP phases in the mixing matrix (see [1]). Therefore, from (2.40), we have

$$\widetilde{\Lambda}_\mu^{fi}(q) \xrightarrow{\text{CP}} \eta_f \eta_i \gamma^0 \mathcal{C} \left[\widetilde{\Lambda}_\mu^{if}(q_{\text{P}}) \right]^T \mathcal{C}^\dagger \gamma^0. \tag{2.67}$$

Imposing a CP constraint analogous to that in (2.24), we obtain

$$\text{CP} \Longleftrightarrow \begin{cases} f_\Omega^{fi} = \eta_f \eta_i f_\Omega^{if} = \eta_f \eta_i \left(f_\Omega^{fi} \right)^*, \\ f_E^{fi} = -\eta_f \eta_i f_E^{if} = -\eta_f \eta_i \left(f_E^{fi} \right)^*, \end{cases} \tag{2.68}$$

with $\Omega = Q, M, A$. Taking into account the constraints (2.61) and (2.62), we have two cases:

$$\text{CP}, \eta_f = \eta_i \Longleftrightarrow f_Q^{fi} = f_M^{fi} = 0, \tag{2.69}$$

$$\text{CP}, \eta_f = -\eta_i \Longleftrightarrow f_E^{fi} = f_A^{fi} = 0. \tag{2.70}$$

Therefore, if CP is conserved, two massive Majorana neutrinos can have either a transition electric form factor or a transition magnetic form factor, but not both, and the transition electric form factor can exist only together with a transition anapole form factor, whereas the transition magnetic form factor can exist only together with a transition charge form factor. In the diagonal case $f = i$, (2.69) does not give any constraint, because only diagonal anapole form factors are allowed for Majorana neutrinos.

2.3. Form Factors in Gauge Models

From the demand that the form factors at zero momentum transfer, $q^2 = 0$, are elements of the scattering matrix, it follows that in any consistent theoretical model the form factors in the matrix element (2.7) should be gauge independent and finite. Then, the form factors values at $q^2 = 0$ determine the static electromagnetic properties of the neutrino that can be probed or measured in the direct interaction with external electromagnetic fields. This is the case for charge, dipole magnetic, and electric neutrino form factors in the minimally extended Standard Model.

In non-Abelian gauge theories, the form factors in the matrix element (2.7) at nonzero momentum transfer, $q^2 \neq 0$, can be noninvariant under gauge transformations. This happens because in general the off-shell photon propagator is gauge dependent. Therefore, the one-photon approximation is not enough to get physical quantities. In this case, the form factors in the matrix element (2.7) cannot be directly measured in an experiment with an external electromagnetic field. However, they can contribute to higher-order diagrams describing some processes that are accessible for experimental observation (see [31]).

Note that there is an important difference between the electromagnetic vertex function of massive and massless neutrinos [32, 33]. For the case of a massless neutrino, the matrix element (2.7) of the electromagnetic current can be expressed in terms of only one Dirac form factor $f_D(q^2)$ (see also [30]):

$$\overline{u}(p')\Lambda_\mu(q)u(p) = f_D\left(q^2\right)\overline{u}(p')\gamma_\mu(1+\gamma_5)u(p). \tag{2.71}$$

It follows that the electric charge and anapole form factors for a massless neutrino are related to the Dirac form factor $f_D(q^2)$, and hence to each other:

$$f_Q\left(q^2\right) = f_D\left(q^2\right), \qquad f_A\left(q^2\right) = \frac{f_D(q^2)}{q^2}. \tag{2.72}$$

In the case of a massive neutrino, there is no such simple relation between electric charge and anapole form factors since the $q_\mu \not{q}\gamma_5$ term in the anapole part of the vertex function (2.18) cannot be neglected.

Moreover, a direct calculation of the massive neutrino electromagnetic vertex function, taking into account all the diagrams in Figures 15–17 of [13], reveals that each of the Feynman diagrams gives nonzero contribution to the term proportional to $\gamma_\mu\gamma_5$ [32, 33]. These contributions are not vanishing even at $q^2 = 0$. Therefore, in addition to the usual four terms in (2.18) an extra term proportional to $\gamma_\mu\gamma_5$ appears and a corresponding additional form factor $f_5(q^2)$ must be introduced. This problem is related to the decomposition of the massive neutrino electromagnetic vertex function. The calculation of the contributions of the proper vertex diagrams (Figure 15 of [13]) and $\gamma - Z$ self-energy diagrams (Figures 16 and 17 of [13]) for arbitrary gauge fixing parameter $\alpha = 1/\xi$ in the general R_ξ gauge and arbitrary mass parameter $a = m_l^2/m_W^2$ shows that at least in the zeroth and first orders of the expansion over the small neutrino mass parameter $b = (m_\nu/m_W)^2$, the corresponding "charge" $f_5(q^2 = 0)$ is zero. The cancellation of contributions from the proper vertex and self-energy diagrams to the form factor $f_5(q^2)$ at $q^2 \neq 0$,

$$f_5\left(q^2\right) = f_5^{(\gamma-Z)}\left(q^2\right) + f_5^{(\text{prop.vert.})}\left(q^2\right) = 0, \tag{2.73}$$

was also shown [32, 33] for arbitrary mass parameters a and b in the 't Hooft-Feynman gauge $\alpha = 1$.

Hence, in the minimally extended Standard Model one can perform a direct calculation of the neutrino vertex function leading to the four terms in (2.18) with gauge-invariant electric charge, magnetic, electric, and anapole moments.

3. Magnetic and Electric Dipole Moments

The neutrino dipole magnetic and electric form factors (and the corresponding magnetic and electric dipole moments) are theoretically the most well-studied and understood among the form factors. They also attract a reasonable attention from experimentalists, although the neutrino magnetic moment predicted in the extended Standard Model with right-handed neutrinos is proportional to the neutrino mass and, therefore, it is many orders of magnitude smaller than the present experimental limits obtained in terrestrial experiments.

3.1. *Theoretical Predictions*

The first calculations of the neutrino dipole moments within the minimal extension of the Standard Model with right-handed neutrinos were performed in [15–22]. The explicit evaluation of the one-loop contributions to the neutrino dipole moments in the leading approximation over the small parameters $b_i = m_i^2/m_W^2$ (where m_i are the neutrino masses, $i = 1, 2, 3$), that in addition exactly accounts for the dependence on the small parameters $a_l = m_l^2/m_W^2$ (with $l = e, \mu, \tau$), yields, for Dirac neutrinos [7, 18–23],

$$\left.\begin{matrix}\mu_{ij}^{\mathrm{D}}\\ \epsilon_{ij}^{\mathrm{D}}\end{matrix}\right\} = \frac{eG_F}{8\sqrt{2}\pi^2}\left(m_i \pm m_j\right)\sum_{l=e,\mu,\tau} f(a_l)U_{li}^{*}U_{lj}, \tag{3.1}$$

where

$$f(a_l) = \frac{3}{4}\left[1 + \frac{1}{1-a_l} - \frac{2a_l}{(1-a_l)^2} - \frac{2a_l^2\ln a_l}{(1-a_l)^3}\right]. \tag{3.2}$$

All the charged lepton parameters a_l are small. In the limit $a_l \ll 1$, one has

$$f(a_l) \simeq \frac{3}{2}\left(1 - \frac{a_l}{2}\right). \tag{3.3}$$

From (3.1) and (3.3), the diagonal magnetic moments of Dirac neutrinos are given by

$$\mu_{ii}^{\mathrm{D}} \simeq \frac{3eG_F m_i}{8\sqrt{2}\pi^2}\left(1 - \frac{1}{2}\sum_{l=e,\mu,\tau} a_l |U_{li}|^2\right). \tag{3.4}$$

This result exhibits the following important features. The magnetic moment of a Dirac neutrino is proportional to the neutrino mass and for a massless Dirac neutrino in the Standard Model (in the absence of right-handed charged currents), the magnetic moment is zero. The magnetic moment of a massive Dirac neutrino, at the leading order in a_l, is independent of the neutrino mixing matrix and of the values of the charged lepton masses. The numerical value of the Dirac neutrino magnetic moment is

$$\mu_{ii}^{\mathrm{D}} \simeq 3.2 \times 10^{-19}\left(\frac{m_i}{\mathrm{eV}}\right)\mu_B. \tag{3.5}$$

Taking into account the existing constraint of the order of 1 eV on the neutrino masses (see [1–4]), this value is several orders of magnitude smaller than the present experimental limits, which are discussed in Section 3.4.

From (3.1), it can be clearly seen that in the extended Standard Model with right-handed neutrinos the static (diagonal) electric dipole moment of a Dirac neutrino vanishes, $\epsilon_{ii}^{\mathrm{D}} = 0$, in spite of possible CP violations generated by the Dirac phase in the mixing matrix (as shown in (2.27), Dirac neutrinos may have nonzero diagonal electric moments only in theories where CP invariance is violated). For a Majorana neutrino, both the diagonal magnetic and electric moments are zero, $\mu_{ii}^{M} = \epsilon_{ii}^{M} = 0$, as shown in Section 2.2.

Let us consider now the neutrino transition moments, which are given by (3.1) for $i \neq j$. Considering only the leading term $f(a_l) \simeq 3/2$ in the expansion (3.3), one gets vanishing transition moments, because of the unitarity relation:

$$\sum_l U_{li}^* U_{lj} = \delta_{ij}. \tag{3.6}$$

Therefore, the first nonvanishing contribution comes from the second term in the expansion (3.3) of $f(a_l)$, which contains the additional small factor $a_l = m_l^2/m_W^2$:

$$\left.\begin{matrix} \mu_{ij}^{\mathrm{D}} \\ \epsilon_{ij}^{\mathrm{D}} \end{matrix}\right\} \simeq -\frac{3eG_F}{32\sqrt{2}\pi^2}(m_i \pm m_j) \sum_{l=e,\mu,\tau} \left(\frac{m_l}{m_W}\right)^2 U_{li}^* U_{lj}, \tag{3.7}$$

for $i \neq j$. Thus, the transition moments are suppressed with respect to the diagonal magnetic moments in (3.4). This suppression is called "GIM mechanism," in analogy with the suppression of flavor-changing neutral currents in hadronic processes discovered in [34]. Numerically, the Dirac transition moments are given by

$$\left.\begin{matrix} \mu_{ij}^{\mathrm{D}} \\ \epsilon_{ij}^{\mathrm{D}} \end{matrix}\right\} \simeq -4 \times 10^{-23} \left(\frac{m_i \pm m_j}{\mathrm{eV}}\right) f_{ij} \mu_B, \tag{3.8}$$

with

$$f_{ij} = \sum_{l=e,\mu,\tau} \left(\frac{m_l}{m_\tau}\right)^2 U_{li}^* U_{lj}. \tag{3.9}$$

Also, Majorana neutrinos can have nonvanishing transition magnetic and electric moments, as discussed in Section 2.2. Assuming CP conservation and neglecting model-dependent Feynman diagrams depending on the details of the scalar sector [7, 21, 22, 24], if ν_i and ν_j have the same CP phase,

$$\mu_{ij}^{M} = 0, \qquad \epsilon_{ij}^{M} = 2\epsilon_{ij}^{\mathrm{D}}, \tag{3.10}$$

whereas if ν_i and ν_j have opposite CP phases,

$$\mu_{ij}^{M} = 2\mu_{ij}^{\mathrm{D}}, \qquad \epsilon_{ij}^{\mathrm{D}} = 0, \tag{3.11}$$

with $\epsilon_{ij}^{\mathrm{D}}$ and μ_{ij}^{D} given by (3.1). Hence, although the nonvanishing Majorana transition moments are twice the Dirac ones, they are equally suppressed by the GIM mechanism. However, the model-dependent contributions of the scalar sector can enhance the Majorana transition moments (see [21, 35, 36]).

In recent studies, the value of the diagonal magnetic moment of a massive Dirac neutrino was calculated in the one-loop approximation in the extended Standard Model with right-handed neutrinos, accounting for the dependence on the neutrino mass parameter $b_i = m_i^2/m_W^2$ [37] and accounting for the exact dependence on both mass parameters b_i and $a_l = m_l^2/m_W^2$ [32, 33]. The calculations of the neutrino magnetic moment which take into account exactly the dependence on the masses of all particles can be useful in the case of a heavy neutrino with a mass comparable or even exceeding the values of the masses of other known particles. Note that the LEP data require that the number of light neutrinos coupled to the Z boson is three [38]. Therefore, any additional active neutrino must be heavier than $m_Z/2$. This possibility is not excluded by current data (see [39]).

For a heavy neutrino with mass m_i, much larger than the charged lepton masses but smaller than the W-boson mass (2 GeV $\ll m_i \ll$ 80 GeV), the authors of [32, 33] obtained the diagonal magnetic moment:

$$\mu_{ii} \simeq \frac{3eG_F}{8\pi^2\sqrt{2}} m_i \left(1 + \frac{5}{18} b_i\right), \tag{3.12}$$

whereas for a heavy neutrino with mass m_i much larger than the W-boson mass, they got

$$\mu_{ii} \simeq \frac{eG_F}{8\pi^2\sqrt{2}} m_i. \tag{3.13}$$

Note that in both cases the Dirac neutrino magnetic moment is proportional to the neutrino mass. This is an expected result, because the calculations have been performed within the extended Standard Model with right-handed neutrinos.

At this point, a question arises: "Is a neutrino magnetic moment always proportional to the neutrino mass?." The answer is "No." For example, much larger values of the Dirac neutrino magnetic moment can be obtained in $SU(2)_L \times SU(2)_R \times U(1)$ left-right symmetric models with direct right-handed neutrino interactions (see [15, 40–42]). The massive gauge bosons states W_1 and W_2 have, respectively, predominant left-handed and right-handed coupling, since

$$W_1 = W_L \cos\xi - W_R \sin\xi, \qquad W_2 = W_L \sin\xi + W_R \cos\xi, \tag{3.14}$$

where ξ is a small mixing angle and the fields W_L and W_R have pure $V \pm A$ interactions. The magnetic moment of a neutrino ν_l calculated in this model, neglecting neutrino mixing, is

$$\mu_{\nu_l} = \frac{eG_F}{2\sqrt{2}\pi^2}\left[m_l\left(1 - \frac{m_{W_1}^2}{m_{W_2}^2}\right)\sin 2\xi + \frac{3}{4}m_{\nu_l}\left(1 + \frac{m_{W_1}^2}{m_{W_2}^2}\right)\right], \tag{3.15}$$

where the term proportional to the charged lepton mass m_l is due to the left-right mixing. This term can exceed the second term in (3.15), which is proportional to the neutrino mass m_{ν_l}.

3.2. Neutrino-Electron Elastic Scattering

The most sensitive and widely used method for the experimental investigation of the neutrino magnetic moment is provided by direct laboratory measurements of low-energy elastic scattering of neutrinos and antineutrinos with electrons in reactor, accelerator and solar experiments. Detailed descriptions of several experiments can be found in [12, 43].

Extensive experimental studies of the neutrino magnetic moment, performed during many years, are stimulated by the hope to observe a value much larger than the prediction in (3.5) of the minimally extended Standard Model with right-handed neutrinos. It would be a clear indication of new physics beyond the extended Standard Model. For example, the effective magnetic moment in $\bar{\nu}_e$-e elastic scattering in a class of extradimension models can be as large as $\sim 10^{-10}\,\mu_B$ [44]. Future higher precision reactor experiments can, therefore, be used to provide new constraints on large extradimensions.

The possibility for neutrino-electron elastic scattering due to neutrino magnetic moment was first considered in [45] and the cross-section of this process was calculated in [46, 47]. Discussions on the derivation of the cross-section and on the optimal conditions for bounding the neutrino magnetic moment, as well as a collection of cross-section formulas for elastic scattering of neutrinos (antineutrinos) on electrons, nucleons, and nuclei can be found in [48, 49].

Let us consider the elastic scattering:

$$\nu + e^- \longrightarrow \nu + e^- \tag{3.16}$$

of a neutrino with energy E_ν with an electron at rest in the laboratory frame. There are two observables: the kinetic energy T of the recoil electron and the recoil angle χ with respect to the neutrino beam, which are related by

$$\cos\chi = \frac{E_\nu + m_e}{E_\nu}\left[\frac{T}{T + 2m_e}\right]^{1/2}. \tag{3.17}$$

The electron kinetic energy is constrained from the energy-momentum conservation by

$$T \leq \frac{2E_\nu^2}{2E_\nu + m_e}. \tag{3.18}$$

Since, in the ultrarelativistic limit, the neutrino magnetic moment interaction changes the neutrino helicity and the Standard Model weak interaction conserves the neutrino helicity, the two contributions add incoherently in the cross-section which can be written as [49]

$$\frac{d\sigma}{dT} = \left(\frac{d\sigma}{dT}\right)_{\text{SM}} + \left(\frac{d\sigma}{dT}\right)_{\mu}. \tag{3.19}$$

The small interference term due to neutrino masses has been derived in [50].

The weak-interaction cross-section is given by

$$\left(\frac{d\sigma}{dT}\right)_{\text{SM}} = \frac{G_F^2 m_e}{2\pi}\left[(g_V + g_A)^2 + (g_V - g_A)^2\left(1 - \frac{T}{E_\nu}\right)^2 + \left(g_{\text{A}}^2 - g_{\text{V}}^2\right)\frac{m_e T}{E_\nu^2}\right], \tag{3.20}$$

with the standard coupling constants g_V and g_A given by

$$g_V = \begin{cases} 2\sin^2\theta_W + \dfrac{1}{2} & \text{for } \nu_e, \\ 2\sin^2\theta_W - \dfrac{1}{2} & \text{for } \nu_\mu, \nu_\tau, \end{cases} \tag{3.21}$$

$$g_A = \begin{cases} \dfrac{1}{2} & \text{for } \nu_e, \\ -\dfrac{1}{2} & \text{for } \nu_\mu, \nu_\tau. \end{cases} \tag{3.22}$$

In antineutrino-electron elastic scattering, one must substitute $g_A \to -g_A$.

The neutrino magnetic-moment contribution to the cross section is given by [49]

$$\left(\frac{d\sigma}{dT}\right)_{\mu} = \frac{\pi\alpha^2}{m_e^2}\left(\frac{1}{T} - \frac{1}{E_\nu}\right)\left(\frac{\mu_\nu}{\mu_B}\right)^2, \tag{3.23}$$

where μ_ν is the effective magnetic moment discussed in Section 3.3.

The two terms $(d\sigma/dT)_{\text{SM}}$ and $(d\sigma/dT)_{\mu}$ exhibit quite different dependences on the experimentally observable electron kinetic energy T, as illustrated in Figure 2, where the values of the two terms averaged over the typical antineutrino reactor spectrum are plotted for six values of the neutrino magnetic moment, $\mu_\nu^{(N)} = N \times 10^{-11}\,\mu_B$, with $N = 1, 2, 3, 4, 5, 6$ [43] (see also [49]). One can see that small values of the neutrino magnetic moment can be probed by lowering the electron recoil energy threshold. In fact, from (3.20) and (3.23), one can find that $(d\sigma/dT)_{\mu}$ exceeds $(d\sigma/dT)_{\text{SM}}$ for

$$T \lesssim \frac{\pi^2\alpha^2}{G_F^2 m_e^3}\left(\frac{\mu_\nu}{\mu_B}\right)^2. \tag{3.24}$$

It was proposed in [51] that electron binding in atoms (the "atomic ionization" effect in neutrino interactions on Ge target) can significantly increase the electromagnetic contribution

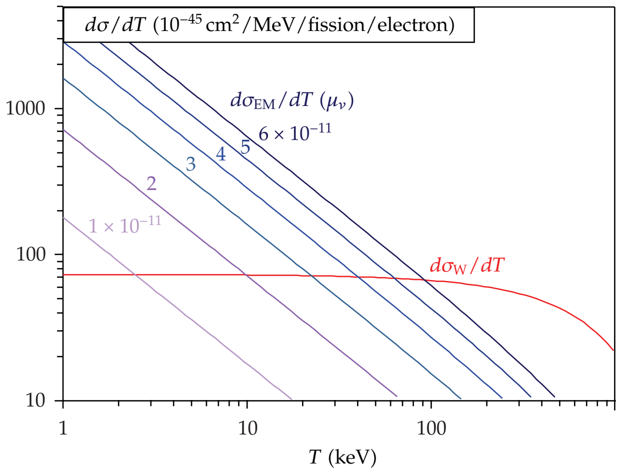

Figure 2: Standard Model weak (W) and magnetic moment electromagnetic (EM) contributions to the cross section for several values of the neutrino magnetic moment [43].

to the differential cross-section with respect to the free electron approximation. However, detailed considerations of the atomic ionization effect in (anti)neutrino atomic electron scattering experiments presented in [52–57] show that the effect is by far too small to have measurable consequences even in the case of the low energy threshold of 2.8 keV reached in the GEMMA experiment [58].

3.3. *Effective Dipole Moments*

In scattering experiments, the neutrino is created at some distance from the detector as a flavor neutrino, which is a superposition of massive neutrinos. Therefore, the magnetic moment that is measured in these experiment is not that of a massive neutrino, but it is an effective magnetic moment which takes into account neutrino mixing and the oscillations during the propagation between source and detector [50, 59]:

$$\mu_\nu^2(\nu_\alpha, L, E) = \sum_j \left| \sum_k U_{\alpha k}^* e^{-i m_k^2 L/2E} \left(\mu_{jk} - i\epsilon_{jk}\right) \right|^2, \tag{3.25}$$

where we have written explicitly the dependence from the initial neutrino flavor ν_α, the distance L, and the energy E. In this expression of the effective μ_ν one can see that in general both the magnetic and electric dipole moments contribute to the elastic scattering. Note that $\mu_\nu(\nu_\alpha, L, E)$ depends only on the neutrino squared-mass differences: considering for simplicity only the magnetic moment contribution, we have

$$\mu_\nu^2(\nu_\alpha, L, E) = \sum_j \sum_{kk'} U_{\alpha k}^* U_{\alpha k'} e^{-i\Delta m_{kk'}^2 L/2E} \mu_{jk}\mu_{jk'}. \tag{3.26}$$

In the case of Majorana neutrinos, there are no diagonal magnetic and electric dipole moments and μ_ν^2 receives contributions only from the transition dipole moments.

Furthermore, if CP is conserved, there are either only magnetic or electric transition dipole moments (see Section 2.2).

The general expression for $\mu_\nu^2(\nu_\alpha, L, E)$ can be simplified in some cases [59]. For instance, for Dirac neutrinos with only diagonal magnetic moments $\mu_{ij} = \mu_i \delta_{ij}$, we have the effective flavor magnetic moment:

$$\mu_\nu^2(\nu_\alpha, L, E) \longrightarrow \left(\mu_\alpha^{\mathrm{D}}\right)^2 = \sum_i |U_{\alpha i}|^2 \mu_i^2. \tag{3.27}$$

Since in this case there is no dependence on the distance L and the neutrino energy, the magnetic cross-section is characterized by the initial neutrino flavor rather than by the composition of mass states in the detector. In this case, measurements of all flavor magnetic moments and mixing parameters can allow the extraction of all the fundamental moments μ_i.

3.4. Experimental Limits

The constraints on the neutrino magnetic moment in direct laboratory experiments have been obtained so far from the lack of any observable distortion of the recoil electron energy spectrum. Experiments of this type have started about 40 years ago. The strategy applied during all these years in reactor experiments is rather simple: minimize the threshold on the recoil energy for the detection of the scattered electron, keeping at the same time a reasonable background level. Since the region of interest coincides with the energy region dominated by radioactivity, low intrinsic radioactivity detectors have been employed together with active and passive shields. In addition, all the experiments were running inside laboratories with significant overburden of concrete, corresponding to several meters of water. This was enough to suppress the soft component of cosmic rays.

In all the experiments, with one exception only, the signal due to the antineutrinos from the reactor (about $2\times10^{20}\,\mathrm{s}^{-1}\,\mathrm{GW\,thermal}^{-1}$) is obtained from the difference between the reactor-on and reactor-off rate. Clearly, this requires the same background with the reactor on as with the reactor off.

$\overline{\nu}_e$-e elastic scattering was first observed in the pioneering experiment [60] at the Savannah River Laboratory. The setup was made of a 15.9 kg plastic scintillator target divided into 16 optically isolated elements, totally enclosed inside a 300 kg NaI crystal shielded with lead and cadmium. Finally, the entire setup was immersed into 2200 liters of liquid scintillator. Both the NaI and the liquid scintillator detectors were working as veto against cosmics and gamma-rays from the laboratory. In the electron kinetic energy range from 1.5 MeV to 4.5 MeV, a reactor-on rate of 47.5 ± 1 events/day was measured, to be compared with a reactor-off rate of 40.4 ± 0.9 events/day. A revised analysis of the Savannah River Laboratory data [49] with an improved reactor neutrino spectrum and a more precise value of $\sin^2\theta_W$ gave hints for a neutrino magnetic moment on the order of $(2–4)\times10^{-10}\,\mu_B$.

However, lower limits were then obtained by two experiments performed at nuclear reactors in Russia. The Krasnoyarsk experiment [61] had a 103 kg target of liquid organofluoric scintillator contained into seven scintillation chambers. The absence of hydrogen in the target was a significant improvement, since the $\overline{\nu}_e$-p charged-current reaction, which has a much larger cross-section than $\overline{\nu}_e$-e, had been an important background source in the Savannah River experiment. The active target was then surrounded by a passive shield of steel, copper, lead, and borated polyethylene. Finally, two layers of plastic scintillators were vetoing the cosmic muons. With a count rate of 8.3 ± 0.3 (reactor-on) and 7.1 ± 0.4 (reactor-off) in the electron energy range 3.15–5.18 MeV, the Krasnoyarsk collaboration obtained the limit:

$$\mu_{\overline{\nu}_e} \leq 2.4 \times 10^{-10}\, \mu_B \quad (90\%\ \text{C.L.}). \tag{3.28}$$

A completely different detector was built for the Rovno [62] experiment: 600 silicon detectors, for a total mass of 75 kg, with a passive shield of mercury, copper, cadmium absorber, and graphite. Finally, the setup was enclosed inside a veto made of plastic scintillators. With a rate of 4963 ± 12 events/day (reactor-on) and 4921 ± 16 events/day (reactor-off) in the electron recoil energy range 0.6–2 MeV, it has been possible to obtain a limit on the neutrino magnetic moment of

$$\mu_{\overline{\nu}_e} \leq 1.9 \times 10^{-10}\, \mu_B \quad (90\%\ \text{C.L.}). \tag{3.29}$$

Finally, more stringent limits have been obtained in the two most recent experiments at rectors. TEXONO [63] has been performed at the Kuo-Sheng nuclear power station. The detector, a 1.06 kg high-purity germanium, was completely surrounded by NaI (Tl) and CsI (Tl) crystals working as anti-Compton. The whole setup was contained inside a shield made of copper, boron-loaded polyethylene, stainless steel, lead, and plastic scintillators. A background of about 1 event/keV·kg·day could be achieved above the threshold of 12 keV, giving the limit:

$$\mu_{\overline{\nu}_e} \leq 7.4 \times 10^{-11}\, \mu_B \quad (90\%\ \text{C.L.}). \tag{3.30}$$

At the moment, the world best limit is coming from the GEMMA experiment at the Kalinin nuclear power plant. A 1.5 kg high-purity germanium detector is placed inside a cup-shaped NaI crystal and surrounded by copper, lead, and plastic scintillators. With an energy threshold as low as 2.8 keV, the GEMMA collaboration obtained [58]

$$\mu_{\overline{\nu}_e} \leq 2.9 \times 10^{-11}\, \mu_B \quad (90\%\ \text{C.L.}). \tag{3.31}$$

The experiment which followed a strategy different from the study of the reactor-on and reactor-off rate was MUNU. As a matter of fact, the detector [64] was able to provide not only the energy but also the topology of events. As a consequence, the initial direction of an electron track could be measured and the electron scattering angle reconstructed. This allowed to look for the reactor signal by comparing forward electrons, having as reference the reactor to detector axis, with the backward ones. In this way, the background is measured online, which eliminates problems from detector instabilities, as well as from a possible time dependence of the background itself. The central component of the detector consisted of an acrylic vessel time-projected chamber (a cylinder 90 cm in diameter and 162 cm long) filled with CF_4 at 3 bar pressure and immersed in a steel tank (2 m diameter and 3.8 m long) filled with 10 m^3 liquid scintillator viewed by 48 photomultipliers. The total target mass of CF_4 was 11.4 kg. Finally, the setup was surrounded by boron-loaded polyethylene and lead. With a total rate of 6.8 ± 0.3 events/day in the forward direction and a background of 5.8 ± 0.17 events/day, the following upper bounds have been obtained [65]:

$$\mu_{\bar{\nu}_e} \leq 9 \times 10^{-11}\, \mu_B \quad (90\%\ \text{C.L.}). \tag{3.32}$$

Several experiments at accelerators have searched for an effect due to the magnetic moment of ν_μ in ν_μ–e and $\bar{\nu}_\mu$–e elastic scattering (see [66]). The current best limit has been obtained in the LSND experiment [67]:

$$\mu_{\nu_\mu} \leq 6.8 \times 10^{-10}\, \mu_B \quad (90\%\ \text{C.L.}). \tag{3.33}$$

The DONUT collaboration has investigated ν_τ–e and $\bar{\nu}_\tau$–e elastic scattering, finding the limit [68]:

$$\mu_{\nu_\tau} \leq 3.9 \times 10^{-7}\, \mu_B \quad (90\%\ \text{C.L.}). \tag{3.34}$$

Solar neutrino experiments as Super-Kamiokande and Borexino can also search for a neutrino magnetic moment signal by studying the shape of the electron spectrum. Since the neutrino magnetic moment depends both on the mixing and on the propagation properties of the neutrino, then oscillations are here relevant.

The analysis of the recoil electron spectrum generated by solar neutrinos in the Super-Kamiokande experiment experiment gave [69]

$$\mu_{\nu} \leq 1.1 \times 10^{-10}\, \mu_B \quad (90\%\ \text{C.L.}), \tag{3.35}$$

where μ_ν is not the same as $\mu_{\bar{\nu}_e}$ since it is given by a different combination of the magnetic moment of the neutrino mass eigenstates (see Section 3.3).

The limit

$$\mu_{\nu} \leq 5.4 \times 10^{-11}\, \mu_B \quad (90\%\ \text{C.L.}) \tag{3.36}$$

has been recently obtained in the Borexino solar neutrino scattering experiment [70]. An upper limit on the neutrino magnetic moment $\mu_\nu \leq 8.4 \times 10^{-11}\, \mu_B$ has been found in an independent analysis of the first release of the Borexino experiment data performed in [71]. It was also shown that with reasonable assumptions on the oscillation probability this limit translates into the conservative upper limits on the magnetic moments of ν_μ and ν_τ:

$$\mu_{\nu_\mu} \leq 1.5 \times 10^{-10}\, \mu_B, \qquad \mu_{\nu_\tau} \leq 1.9 \times 10^{-10}\, \mu_B \quad (90\%\ \text{C.L.}). \tag{3.37}$$

The limit on μ_{ν_τ} is three order of magnitude stronger than the direct limit (3.34).

The global fit [72, 73] of the magnetic moment data from the reactor and solar neutrino experiments for the Majorana neutrinos produces limits on the neutrino transition moments:

$$\mu_{23}, \mu_{31}, \mu_{12} < 1.8 \times 10^{-10}\, \mu_B \quad (90\%\ \text{C.L.}). \tag{3.38}$$

Finally, an interesting new possibility for providing more stringent constraints on the neutrino magnetic moment from $\overline{\nu}_e$-e scattering experiments was discussed in [74] on the basis of an observation [75] that "dynamical zeros" appear in the Standard Model contribution to the scattering cross-section.

3.5. Theoretical Considerations

As it was already mentioned before, there is a gap of many orders of magnitude between the present experimental limits $\sim 10^{-11}\,\mu_B$ on neutrino magnetic moments (discussed in Section 3.4) and the prediction (3.5) of the minimal extension of the Standard Model with right-handed neutrinos. At the same time, the experimental sensitivity of reactor $\overline{\nu}_e$-e elastic scattering experiments has improved by only one order of magnitude during a period of about twenty years (see [49], where a sensitivity of $\sim 10^{-10}\,\mu_B$ is discussed). However, the experimental studies of neutrino magnetic moments are stimulated by the hope that new physics beyond the minimally extended Standard Model with right-handed neutrinos might give much stronger contributions. One of the examples in which it is possible to avoid the neutrino magnetic moment being proportional to a (small) neutrino mass, that would in principle make a neutrino magnetic moment accessible for experimental observations, is realized in the left-right symmetric models considered at the end of Section 3.1.

Other interesting possibilities of obtaining neutrino magnetic moments lager than the prediction (3.5) of the minimal extension the Standard Model with right-handed neutrinos have been considered recently. In this concern, we note that it was proposed in [44] to probe a class of large extradimensions models with future reactors searches for neutrino magnetic moments. The results obtained within the Minimal Supersymmetric Standard Model with R-parity violating interactions [76, 77] show that the Majorana transition magnetic moment might be significantly above the scale of (3.5).

Considering the problem of large neutrino magnetic moments, one can write down a generic relation between the size of a neutrino magnetic moment μ_ν and the corresponding neutrino mass m_ν [28, 29, 35, 78–80]. Suppose that a large neutrino magnetic moment is generated by physics beyond a minimal extension of the Standard Model at an energy scale characterized by Λ. For a generic diagram corresponding to this contribution to μ_ν, one can again use the Feynman graph in Figure 1(b); the shaded circle in this case denotes effects of new physics beyond the Standard Model. The contribution of this diagram to the magnetic moment is

$$\mu_\nu \sim \frac{eG}{\Lambda}, \tag{3.39}$$

where e is the electric charge and G is a combination of coupling constants and loop factors. The same diagram of Figure 1(b) but without the photon line gives a new physics contribution to the neutrino mass:

$$\delta m_\nu \sim G\Lambda. \tag{3.40}$$

Combining the estimates (3.39) and (3.40), one can get the relation:

$$\delta m_\nu \sim \frac{\Lambda^2}{2m_e}\frac{\mu_\nu}{\mu_B} = \frac{\mu_\nu}{10^{-18}\mu_B}\left(\frac{\Lambda}{1\ \text{TeV}}\right)^2 \text{eV} \tag{3.41}$$

between the one-loop contribution to the neutrino mass and the neutrino magnetic moment.

It follows that, generally, in theoretical models that predict large values for the neutrino magnetic moment, simultaneously large contributions to the neutrino mass arise. Therefore, a particular fine tuning is needed to get a large value for the neutrino magnetic moment while keeping the neutrino mass within experimental bounds. One of the possibilities [78] is based on the idea of suppressing the ratio m_ν/μ_ν with a symmetry: if a $SU(2)_\nu$ symmetry is an exact symmetry of the Lagrangian of a model, because of different symmetry properties of the mass and magnetic moment even a massless neutrino can have a nonzero magnetic moment. If, as it happens in a realistic model, the $SU(2)_\nu$ symmetry is broken and if this breaking is small, the ratio m_ν/μ_ν is also small, giving a natural way to obtain a magnetic moment of the order of $\sim 10^{-11}\mu_B$ without contradictions with the neutrino mass experimental constraints. Several possibilities based on the general idea of [78] were considered in [81–86].

Another idea of neutrino mass suppression without suppression of the neutrino magnetic moment was discussed in [35] within the Zee model [87], which is based on the Standard Model gauge group $SU(2)_L \times U(1)_Y$ and contains at least three Higgs doublets and a charged field which is a singlet of $SU(2)_L$. For this kind of models, there is a suppression of the neutrino mass diagram, while the magnetic moment diagram is not suppressed.

It is possible to show with more general and rigorous considerations [28, 29, 80] that the Λ^2 dependence in (3.41) arises from the quadratic divergence in the renormalization of the dimension-four neutrino mass operator. A general and model-independent upper bound on the Dirac neutrino magnetic moment, which can be generated by an effective theory beyond the Standard Model, has been derived [28, 29, 80] from the demand of absence of fine-tuning of effective operator coefficients and from the current experimental information on neutrino masses. A model with Dirac fermions, scalars, and gauge bosons that is valid below the scale Λ and respects the Standard Model $SU(2)_L\times U(1)_Y$ symmetry was considered. Integrating out the physics above the scale Λ, the following effective Lagrangian that involves right-handed neutrinos ν_R, lepton isodoublets and the Higgs doublet can be obtained:

$$\mathcal{L}_{\text{eff}} = \sum_{n,j} \frac{C_j^n(\mu)}{\Lambda^{n-4}} \mathcal{O}_j^{(n)}(\mu) + \text{H.c.}, \tag{3.42}$$

where μ is the renormalization scale, $n \geq 4$ denotes the operator dimension and j runs over independent operators of a given dimension. For $n = 4$, a neutrino mass arises from the operator $\mathcal{O}_1^{(4)} = \overline{L}\widetilde{\Phi}\nu_R$, where $\widetilde{\Phi} = i\sigma_2\Phi^*$. In addition, if the scale Λ is not extremely large with respect to the electroweak scale, an important contribution to the neutrino mass can arise also from higher dimension operators. At this point, it is important to note that the combination of the $n = 6$ operators appearing in the Lagrangian (3.42) contains the magnetic moment operator $\overline{\nu}\sigma_{\mu\nu}\nu F^{\mu\nu}$ and also generates a contribution δm_ν to the neutrino mass [28, 29, 80]. Solving the renormalization group equation from the scale Λ to the electroweak scale, one finds that the contributions to the neutrino magnetic moment and to the neutrino mass are connected to each other by

$$\left|\mu_\nu^{\mathrm{D}}\right| = \frac{16\sqrt{2}G_F m_e \delta m_\nu \sin^4\theta_W}{9\alpha^2 \left|f\right| \ln(\Lambda/v)}\mu_B, \tag{3.43}$$

where α is the fine structure constant, v is the vacuum expectation value of the Higgs doublet,

$$f = 1 - r - \frac{2}{3}\tan^2\theta_W - \frac{1}{3}(1+r)\tan^4\theta_W, \tag{3.44}$$

and r is a ratio of effective operator coefficients defined at the scale Λ which is of order unity without fine tuning. If the neutrino magnetic moment is generated by new physics at a scale $\Lambda \sim 1\,\mathrm{TeV}$ and the corresponding contribution to the neutrino mass is $\delta m_\nu \lesssim 1\,\mathrm{eV}$, then the bound $\mu_\nu \lesssim 10^{-14}\,\mu_B$ can be obtained. This bound is some orders of magnitude stronger than the constraints from reactor and solar neutrino scattering experiments discussed before.

The model-independent limit on a Majorana neutrino transition magnetic moment μ_ν^{M} was also discussed in [28, 29, 80]. However, the limit in the Majorana case is much weaker than that in the Dirac case, because for a Majorana neutrino, the magnetic moment contribution to the mass is Yukawa suppressed. The limit on μ_ν^{M} is also weaker than the present experimental limits if μ_ν^{M} is generated by new physics at the scale $\Lambda \sim 1$ TeV. An important conclusion of [28, 29, 80], based on model-independent considerations of the contributions to μ_ν, is that if a neutrino magnetic moment of order $\mu_\nu \geq 10^{-15}\,\mu_B$ were observed in an experiment, it would give a proof that neutrinos are Majorana rather than Dirac particles.

4. Neutrino Charge Radius

Even if the electric charge of a neutrino is vanishing, the electric form factor $f_Q(q^2)$ can still contain nontrivial information about neutrino electromagnetic properties. Considering $f_Q(0) = 0$, in the static limit ($q^2 \to 0$), the electric form factor is given by

$$f_Q\left(q^2\right) = q^2 \left.\frac{df_Q(q^2)}{dq^2}\right|_{q^2=0} + \cdots . \tag{4.1}$$

The leading contribution can be expressed in terms of a neutrino charge radius considering a static spherically symmetric charge distribution of density $\rho(r)$ (with $r = |\vec{x}|$) in the so-called "Breit frame," where $q_0 = 0$. In this approximation, we have

$$f_Q\left(q^2\right) = \int \rho(r) e^{i\vec{q}\cdot\vec{x}} d^3x = 4\pi \int \rho(r) \frac{\sin(qr)}{qr} r^2 dr, \tag{4.2}$$

where $q = |\vec{q}|$. Since $df_Q/dq^2|_{q^2=0} = -\langle r^2\rangle/6$, with $\langle r^2\rangle = \int r^2\rho(r)dr$, the neutrino charge radius is defined by

$$\left\langle r_\nu^2 \right\rangle = -6\,\frac{df_Q(q^2)}{dq^2}\bigg|_{q^2=0}. \tag{4.3}$$

Note that $\langle r_\nu^2\rangle$ can be negative, because the charge density $\rho(r)$ is not a positively defined function of r.

In one of the first studies [31], it was claimed that in the Standard Model and in the unitary gauge, the neutrino charge radius is ultraviolet-divergent and so it is not a physical quantity. A direct one-loop calculation [32, 33] of proper vertices and $\gamma - Z$ self-energy (Figures 15 and 16 of [13]) contributions to the neutrino charge radius performed in a general R_ξ gauge for a massive Dirac neutrino gave also a divergent result. However, it was shown [88], using the unitary gauge, that, by including in addition to the usual terms also contributions from diagrams of the neutrino-lepton neutral current scattering (Z boson diagrams), it is possible to obtain for the neutrino charge radius a gauge-dependent but finite quantity. Later on, it was also shown [16] that in order to define the neutrino charge radius as a physical quantity, one has also to consider box diagrams (see Figure 18 of [13]), which contribute to the scattering process $\nu + \ell \rightarrow \nu + \ell$, and that in combination with contributions from the proper diagrams it is possible to obtain a finite and gauge-independent value for the neutrino charge radius. In this way, the neutrino electroweak radius was defined [89, 90] and an additional set of diagrams that give contribution to its value was discussed in [91]. Finally, in a series of papers [92–94] the neutrino electroweak radius as a physical observable has been introduced. In the corresponding calculations, performed in the one-loop approximation including additional terms from the $\gamma - Z$ boson mixing and the box diagrams involving W and Z bosons, the following gauge-invariant result for the neutrino charge radius has been obtained:

$$\left\langle r_{\nu_\alpha}^2 \right\rangle = \frac{G_F}{4\sqrt{2}\pi^2}\left[3 - 2\log\left(\frac{m_\alpha^2}{m_W^2}\right)\right], \tag{4.4}$$

where m_W and m_α are the W boson and lepton masses ($\alpha = e, \mu, \tau$). This result, however, revived the discussion [95–98] on the definition of the neutrino charge radius. Numerically, for the electron neutrino electroweak radius, it yields [92–94]

$$\left\langle r_{\nu_e}^2 \right\rangle = 4 \times 10^{-33}\ \text{cm}^2, \tag{4.5}$$

which is very close to the numerical estimations obtained much earlier in [89, 90].

Note that the neutrino charge radius can be considered as an effective scale of the particle's "size," which should influence physical processes such as, for instance, neutrino scattering off electron. To incorporate the neutrino charge radius contribution in the cross-section (3.20), the following substitution [49, 99, 100] can be used:

$$g_V \longrightarrow \frac{1}{2} + 2\sin^2\theta_W + \frac{2}{3}m_W^2\left\langle r_{\nu_e}^2 \right\rangle \sin^2\theta_W. \tag{4.6}$$

Using this method, the TEXONO collaboration obtained [101]

$$-2.1 \times 10^{-32}\,\text{cm}^2 < \left\langle r^2_{\bar{\nu}_e} \right\rangle < 3.3 \times 10^{-32}\,\text{cm}^2 \quad (90\%\ \text{C.L.}). \tag{4.7}$$

Other available bounds on the electron neutrino charge radius are from primordial nucleosynthesis [102]

$$\left\langle r^2_{\nu_e} \right\rangle \lesssim 7 \times 10^{-33}\,\text{cm}^2, \tag{4.8}$$

from SN 1987A [103]

$$\left\langle r^2_{\bar{\nu}_e} \right\rangle \lesssim 2 \times 10^{-33}\,\text{cm}^2, \tag{4.9}$$

from neutrino neutral-current reactions [104]

$$-2.74 \times 10^{-32}\,\text{cm}^2 < \left\langle r^2_{\nu_e} \right\rangle < 4.88 \times 10^{-32}\,\text{cm}^2 \quad (90\%\ \text{C.L.}), \tag{4.10}$$

from solar experiments (Kamiokande II and Homestake) [105, 106]

$$\left\langle r^2_{\nu_e} \right\rangle < 2.3 \times 10^{-32}\,\text{cm}^2 \quad (95\%\ \text{C.L.}), \tag{4.11}$$

from an evaluation of the weak mixing angle $\sin^2\theta_W$ by a combined fit of all electron neutrino elastic scattering data [107]

$$-0.13 \times 10^{-32}\,\text{cm}^2 < \left\langle r^2_{\nu_e} \right\rangle < 3.32 \times 10^{-32}\,\text{cm}^2 \quad (90\%\ \text{C.L.}). \tag{4.12}$$

Comparing the theoretical value in (4.5) with the experimental limits in (4.7)–(4.12), one can see that they differ at most by one order of magnitude. Therefore, one may expect that the experimental accuracy will soon reach the value needed to probe the theoretical predictions for the neutrino effective charge radius.

The effects of new physics beyond the Standard Model can also contribute to the neutrino charge radius. Let us only mention that the anomalous $WW\gamma$ vertex contribution to the neutrino effective charge radius has been studied in [30], and shown to correspond to a contribution $\lesssim 10^{-34}\,\text{cm}^2$ to $|\langle r^2_{\nu_e} \rangle|$. Note that this is only one order of magnitude lower than the expected value of the charge radius in the Standard Model.

A detailed discussion on the possibilities to constrain the ν_τ and ν_μ charge radii from astrophysical and cosmological observations and from terrestrial experiments can be found in [108].

5. Radiative Decay and Plasmon Decay

If the masses of neutrinos are nondegenerate, the radiative decay of a heavier neutrino ν_i into a lighter neutrino ν_f (with $m_i > m_f$) with emission of a photon,

$$\nu_i \longrightarrow \nu_f + \gamma, \tag{5.1}$$

may proceed in vacuum [15, 16, 18–21, 23, 109, 110]. Early discussions of the possible role of neutrino radiative decay in different astrophysical and cosmological settings can be found in [111–116].

For the case of a Dirac neutrino, the decay rate in the minimal extension of the Standard Model with right-handed neutrinos is [15, 16, 18–21, 23, 109, 110]

$$\Gamma_{\nu_i^D \to \nu_j^D + \gamma} = \frac{\alpha G_F^2}{128\pi^4}\left(\frac{m_i^2 - m_j^2}{m_j}\right)^3 \left(m_i^2 + m_j^2\right)\left|\sum_{l=e,\mu,\tau} f(a_l) U_{lj} U_{li}^*\right|^2, \tag{5.2}$$

where $f(a_l)$ is given by (3.2). Recalling the results for the Dirac neutrino magnetic and electric transition moments μ_{ij} and ϵ_{ij}, given in (3.1), one can rewrite (5.2) as (see [117, 118])

$$\Gamma_{\nu_i \to \nu_j + \gamma} = \frac{\left|\mu_{ij}\right|^2 + \left|\epsilon_{ij}^2\right|}{8\pi}\left(\frac{m_i^2 - m_j^2}{m_j}\right)^3. \tag{5.3}$$

For degenerate neutrino masses ($m_i = m_j$), the process is kinematically forbidden in vacuum.

Note that there are models (see for instance [119]) in which the neutrino radiative decay rates (as well as the magnetic moment discussed above) of a nonstandard Dirac neutrinos are much larger than those predicted in the minimally extended Standard Model.

For Majorana neutrinos, if CP is violated, the decay rate is given by (5.3). If CP is conserved, we have two cases: if the Majorana neutrinos ν_i and ν_j have the same CP eigenvalues, (2.69) and (3.10) imply that the decay process is induced purely by the neutrino electric transition dipole moment, because $\mu_{ij} = 0$; on the other hand, if the two Majorana neutrinos have opposite CP eigenvalues, from (2.70) and (3.11), one can see that the transition is purely of magnetic dipole type ($\epsilon_{ij} = 0$).

For numerical estimations, it is convenient to express (5.3) in the following form:

$$\Gamma_{\nu_i \to \nu_j + \gamma} = 5.3\left(\frac{\mu_{\text{eff}}}{\mu_B}\right)^2 \left(\frac{m_i^2 - m_j^2}{m_j^2}\right)^3 \left(\frac{m_i}{1\,\text{eV}}\right)^3 s^{-1}, \tag{5.4}$$

with the effective neutrino magnetic moment $\mu_{\text{eff}} = \sqrt{|\mu_{ij}|^2 + |\epsilon_{ij}^2|}$.

The neutrino radiative decay can be constrained by the absence of decay photons in reactor $\overline{\nu}_e$ and solar ν_e fluxes. The limits on μ_{eff} that have been obtained from these considerations are much weaker than those obtained from neutrino scattering terrestrial experiments. Stronger constraints on μ_{eff} (though still weaker than the terrestrial ones) have been obtained from the neutrino decay limit set by SN 1987A and from the limits on the

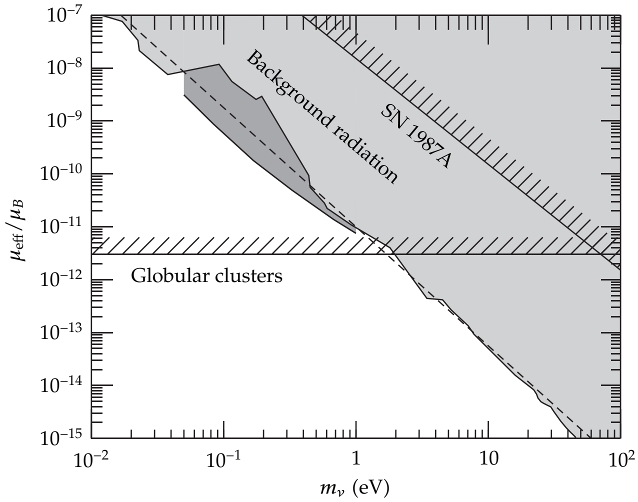

Figure 3: Astrophysical limits on neutrino transition moments [117, 118].

distortion of the Cosmic Microwave Background Radiation (CMBR). These limits can be expressed as (see [117, 118] and references therein)

$$\frac{\mu_{\rm eff}}{\mu_{\rm B}} < \begin{cases} 0.9 \times 10^{-1}\left(\dfrac{\rm eV}{m_\nu}\right)^2 & \text{Reactor } (\overline{\nu}_{\rm e}), \\ 0.5 \times 10^{-5}\left(\dfrac{\rm eV}{m_\nu}\right)^2 & \text{Sun } (\nu_{\rm e}), \\ 1.5 \times 10^{-8}\left(\dfrac{\rm eV}{m_\nu}\right)^2 & \text{SN1987A (all flavors)}, \\ 1.0 \times 10^{-11}\left(\dfrac{\rm eV}{m_\nu}\right)^{9/4} & \text{CMBR (all flavors)}. \end{cases} \tag{5.5}$$

Detailed discussions (and corresponding references) on the astrophysical constraints on the neutrino magnetic and electric transition moments, summarized in Figure 3, can be found in [117, 118].

For completeness, we would like to mention that other processes characterized by the same signature of (5.1) have been considered (for a review of the literature see [120–124]).

(i) The photon radiation by a massless neutrino ($\nu_i \to \nu_j + \gamma, i = j$) due to the vacuum polarization loop diagram in the presence of an external magnetic field [125].

(ii) The photon radiation by a massive neutrino with nonvanishing magnetic moment in constant magnetic and electromagnetic wave fields [126–129].

(iii) The Cherenkov radiation due to the nonvanishing neutrino magnetic moment in an homogeneous and infinitely extended medium, which is only possible if the speed of the neutrino is larger than the speed of light in the medium [130, 131].

(iv) The transition radiation due to a nonvanishing neutrino magnetic moment which would be produced when the neutrino crosses the interface of two media with different refractive indices [132–134].

(v) The Cherenkov radiation of a massless neutrino due to its induced charge in a medium [135, 136]. (Note that the neutrino electromagnetic properties are in general affected by the external environment. In particular, a neutrino can acquire an electric charge in magnetized matter [135, 136] and the neutrino magnetic moment depends on the strength of external electromagnetic fields [127, 137, 138]. A recent study of the neutrino electromagnetic vertex in magnetized matter can be found in [139]. See also [122, 123] for a review of neutrino interactions in external electromagnetic fields.)

(vi) The Cherenkov radiation of massive and massless neutrinos in a magnetized medium [120, 140].

(vii) The neutrino radiative decay ($\nu_i \to \nu_j + \gamma, i \neq j$) in external fields and media (see [141–146] and references therein).

(viii) The spin light of neutrino in matter ($SL\nu$) that is a mechanism of electromagnetic radiation due to the precession or transition of magnetic or electric (transition) moments of massive neutrinos when they propagate in background matter [121, 147–151].

A very interesting process, for the purpose of constraining neutrino electromagnetic properties, is the photon (plasmon) decay into a neutrino-antineutrino pair:

$$\gamma^* \longrightarrow \nu + \overline{\nu}. \tag{5.6}$$

This process becomes kinematically allowed in media, because a photon with the dispersion relation $\omega_\gamma^2 + \vec{k}_\gamma^2 > 0$ roughly behaves as a particle with an effective mass.

Plasmon decay generated by the neutrino coupling to photons due to a magnetic moment μ_ν (and/or to a neutrino electric millicharge q_ν) was first considered in [152] as a possible source of energy loss of the Sun. The requirement that the energy loss does not exceed the solar luminosity gave [117, 118]

$$\mu_\nu \lesssim 4 \times 10^{-10}\ \mu_B, \tag{5.7}$$

and $q_\nu \lesssim 6 \times 10^{-14}\, e$.

The tightest astrophysical bound on a neutrino magnetic moment is provided by the observed properties of globular cluster stars. The plasmon decay (5.6) inside the star liberates the energy ω_γ in the form of neutrinos that freely escape the stellar environment. This nonstandard energy loss cools a red giant star so fast that it can delay helium ignition. From the lack of observational evidence of this effect, the following limit has been found [153]:

$$\mu_\nu \leq 3 \times 10^{-12}\, \mu_B, \tag{5.8}$$

and $q_\nu \lesssim 2 \times 10^{-14}\, e$. This is the most stringent astrophysical constraint on a neutrino magnetic moment, applicable to both Dirac and Majorana neutrinos. The same limit applies for the neutrino magnetic transition moments as well as for the electric (transition) moments.

Recently, it has been shown that the additional cooling due to neutrino magnetic moments generates qualitative changes to the structure and evolution of stars with masses between 7 and 18 solar masses, rather than simply changing the time scales of their burning [154]. The resulting sensitivity to the neutrino magnetic moment has been estimated to be at the level of $(2-4)\times 10^{-11}\ \mu_B$.

6. Spin-Flavor Precession

If neutrinos have magnetic moments, the spin can precess in a transverse magnetic field [155–157].

Let us first consider the spin precession of a Dirac neutrino generated by its diagonal magnetic moment μ. The spatial evolution of the left-handed and right-handed helicity amplitudes $\varphi_L(x)$ and $\varphi_R(x)$ in a transverse magnetic field $B_\perp(x)$ is given by

$$i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\varphi_L(x)\\ \varphi_R(x)\end{pmatrix} = \begin{pmatrix}0 & \mu B_\perp(x)\\ \mu B_\perp(x) & 0\end{pmatrix}\begin{pmatrix}\varphi_L(x)\\ \varphi_R(x)\end{pmatrix}. \tag{6.1}$$

The differential equation (6.1) can be solved through the transformation:

$$\begin{pmatrix}\varphi_L(x)\\ \varphi_R(x)\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}\begin{pmatrix}\varphi_-(x)\\ \varphi_+(x)\end{pmatrix}. \tag{6.2}$$

The new amplitudes $\varphi_-(x)$ and $\varphi_+(x)$ satisfy decoupled differential equations, whose solutions are

$$\varphi_\mp(x) = \exp\left[\pm i\int_0^x \mathrm{d}x'\mu B_\perp(x')\right]\varphi_\mp(0). \tag{6.3}$$

If we consider an initial left-handed neutrino, we have

$$\begin{pmatrix}\varphi_L(0)\\ \varphi_R(0)\end{pmatrix} = \begin{pmatrix}1\\ 0\end{pmatrix} \Longrightarrow \begin{pmatrix}\varphi_-(0)\\ \varphi_+(0)\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix}. \tag{6.4}$$

Then, the probability of $\nu_L \to \nu_R$ transitions is given by

$$P_{\nu_L\to\nu_R}(x) = \left|\varphi_R(x)\right|^2 = \sin^2\left(\int_0^x \mathrm{d}x'\mu B_\perp(x')\right). \tag{6.5}$$

Note that the transition probability is independent from the neutrino energy (contrary to the case of flavor oscillations) and the amplitude of the oscillating probability is unity. Hence, when the argument of the sine is equal to $\pi/2$, there is complete $\nu_L \to \nu_R$ conversion.

The precession $\nu_{eL} \to \nu_{eR}$ in the magnetic field of the Sun was considered in 1971 [155] as a possible solution of the solar neutrino problem. If neutrinos are Dirac particles, right-handed neutrinos are sterile and a $\nu_{eL} \to \nu_{eR}$ conversion could explain the disappearance of active solar ν_{eL}'s.

In 1986, it was realized [156, 157] that the matter effect during neutrino propagation inside of the Sun suppresses $\nu_{eL} \to \nu_{eR}$ transition by lifting the degeneracy of ν_{eL} and ν_{eR}.

Indeed, taking into account matter effects, the evolution equation (6.1) for a Dirac neutrino becomes

$$i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\varphi_L(x)\\ \varphi_R(x)\end{pmatrix} = \begin{pmatrix}V(x) & \mu B_\perp(x)\\ \mu B_\perp(x) & 0\end{pmatrix}\begin{pmatrix}\varphi_L(x)\\ \varphi_R(x)\end{pmatrix}, \tag{6.6}$$

with the appropriate potential $V(x)$ which depends on the neutrino flavor:

$$V_\alpha(x) = V_{\mathrm{CC}}(x)\ \delta_{\alpha e} + V_{\mathrm{NC}}(x). \tag{6.7}$$

Here, V_{CC} and V_{NC} are the charged-current and neutral-current potentials given by

$$V_{\mathrm{CC}}(x) = \sqrt{2}G_F N_e(x), \qquad V_{\mathrm{NC}}(x) = -\frac{1}{2}\sqrt{2}G_F N_n(x), \tag{6.8}$$

where $N_e(x)$ and $N_n(x)$ are the electron and neutron number densities in the medium. For antineutrinos, $\overline{V}_\alpha(x) = -V_\alpha(x)$.

In the case of a constant matter density, the differential equation (6.6) can be solved analytically with the orthogonal transformation:

$$\begin{pmatrix}\varphi_L(x)\\ \varphi_R(x)\end{pmatrix} = \begin{pmatrix}\cos\xi & \sin\xi\\ -\sin\xi & \cos\xi\end{pmatrix}\begin{pmatrix}\varphi_-(x)\\ \varphi_+(x)\end{pmatrix}. \tag{6.9}$$

The angle ξ is chosen in order to diagonalize the matrix operator in (6.6):

$$\sin 2\xi = \frac{2\mu B_\perp}{\Delta E_M}, \tag{6.10}$$

with the effective energy splitting in matter

$$\Delta E_{\mathrm{M}} = \sqrt{V^2 + (2\mu B_\perp)^2}. \tag{6.11}$$

The decoupled evolution of $\varphi_\mp(x)$ is given by:

$$\varphi_\mp(x) = \exp\left[-\frac{i}{2}(V \mp \Delta E_M)\right]\varphi_\mp(0). \tag{6.12}$$

For an initial left-handed neutrino,

$$\begin{pmatrix}\varphi_-(0)\\ \varphi_+(0)\end{pmatrix} = \begin{pmatrix}\cos\xi\\ \sin\xi\end{pmatrix}, \tag{6.13}$$

leading to the oscillatory transition probability:

$$P_{\nu_L\to\nu_R}(x) = |\varphi_R(x)|^2 = \sin^2 2\xi \sin^2\left(\frac{1}{2}\ \Delta E_M x\right). \tag{6.14}$$

Since in matter $\Delta E_M > 2\mu B_\perp$, the matter effect suppresses the amplitude of $\nu_L \to \nu_R$ transitions. However, these transitions are still independent from the neutrino energy, which does not enter in the evolution equation (6.6).

When it was known, in 1986 [156, 157], that the matter potential has the effect of suppressing $\nu_L \to \nu_R$ transitions because it breaks the degeneracy of left-handed and right-handed states, it did not take long to realize, in 1988 [158, 159], that the matter potentials can cause resonant spin-flavor precession if different flavor neutrinos have transition magnetic moments (spin-flavor precession in vacuum was previously discussed in [24]).

Let us denote with $\varphi_{\alpha h}(x)$ the flavor and helicity amplitudes (with $\alpha = e, \mu, \tau$ and $h = \pm 1$), that is, $\varphi_{\alpha -1}(x) \equiv \varphi_{\alpha L}(x)$ and $\varphi_{\alpha +1}(x) \equiv \varphi_{\alpha R}(x)$. Considering neutrino mixing, the evolution of the flavor and helicity amplitudes is given by

$$i\,\frac{\mathrm{d}\varphi_{\alpha h}(x)}{\mathrm{d}x} = \sum_{\beta}\sum_{h'=\pm 1}\left[\left(\sum_k U_{\alpha k}\frac{m_k^2}{2E}U_{\beta k}^* + V_\alpha(x)\delta_{\alpha\beta}\right)\delta_{hh'} + \mu_{\alpha\beta}B_\perp(x)\delta_{-hh'}\right]\varphi_{\beta h'}(x), \tag{6.15}$$

with the effective magnetic moments in the flavor basis:

$$\mu_{\alpha\beta} = \sum_{k,j} U_{\alpha k}\mu_{kj}U_{\beta j}^*. \tag{6.16}$$

For a Dirac neutrino, from (2.36), we have

$$\mu_{jk} = \mu_{kj}{}^* \Longrightarrow \mu_{\beta\alpha} = \mu_{\alpha\beta}^*. \tag{6.17}$$

If CP is conserved, from (2.43), and the reality of the mixing matrix, for a Dirac neutrino, we obtain

$$\mathrm{CP} \Longrightarrow \mu_{jk} = \mu_{kj} \Longrightarrow \mu_{\beta\alpha} = \mu_{\alpha\beta}. \tag{6.18}$$

For a Majorana neutrino, from (2.59) and (2.61), we have

$$\mu_{jk} = -\mu_{\mathrm{k}j}, \qquad \mu_{kj} = -\mu_{kj}{}^*. \tag{6.19}$$

Hence, in the mass basis of Majorana neutrinos, there are no diagonal magnetic moments and the transition magnetic moments are imaginary. If CP is not conserved, the mixing matrix is not real and the constraints (6.19) do not imply similar relations between the effective magnetic moments in the flavor basis, for which we have only the relation in (6.17) as for Dirac neutrinos. In particular, Majorana neutrinos can have diagonal effective magnetic moments in the flavor basis if CP is not conserved. Let us emphasize that both Dirac and Majorana phases contribute to this effect. Therefore, it occurs also in the case of two-neutrino mixing, in which there is one Majorana phase.

On the other hand, if CP is conserved, there is no additional constraint on the magnetic moments of Majorana neutrinos in the mass basis, as we have seen in Section 2.2. However, in this case, the mixing matrix is real and we have

$$\mathrm{CP} \Longrightarrow \mu_{\beta\alpha} = -\mu_{\alpha\beta}, \qquad \mu_{\alpha\beta} = -\mu_{\alpha\beta}{}^*. \tag{6.20}$$

Hence, only if CP is conserved, there are no diagonal magnetic moments of Majorana neutrinos in the flavor basis as in the mass basis.

In the following, we discuss the spin-flavor evolution equation in the two-neutrino mixing approximation, which is interesting for understanding the relevant features of neutrino spin-flavor precession.

Considering Dirac neutrinos, from (6.15), it follows that the generalization of (6.1) to two-neutrino $\nu_e - \nu_\mu$ mixing is, using an analogous notation,

$$i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\varphi_{eL}(x)\\ \varphi_{\mu L}(x)\\ \varphi_{eR}(x)\\ \varphi_{\mu R}(x)\end{pmatrix} = \mathrm{H}\begin{pmatrix}\varphi_{eL}(x)\\ \varphi_{\mu L}(x)\\ \varphi_{eR}(x)\\ \varphi_{\mu R}(x)\end{pmatrix}, \tag{6.21}$$

with the effective Hamiltonian matrix:

$$\mathrm{H} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\vartheta + V_e & \frac{\Delta m^2}{4E}\sin 2\vartheta & \mu_{ee}B_\perp(x) & \mu_{e\mu}B_\perp(x) \\ \frac{\Delta m^2}{4E}\sin 2\vartheta & \frac{\Delta m^2}{4E}\cos 2\vartheta + V_\mu & \mu_{e\mu}^* B_\perp(x) & \mu_{\mu\mu}B_\perp(x) \\ \mu_{ee}B_\perp(x) & \mu_{e\mu}B_\perp(x) & -\frac{\Delta m^2}{4E}\cos 2\vartheta & \frac{\Delta m^2}{4E}\sin 2\vartheta \\ \mu_{e\mu}^* B_\perp(x) & \mu_{\mu\mu}B_\perp(x) & \frac{\Delta m^2}{4E}\sin 2\vartheta & \frac{\Delta m^2}{4E}\cos 2\vartheta \end{pmatrix}, \tag{6.22}$$

where we have used the constraint (6.17) for the transition magnetic moments. The matter potential can generate resonances, which occur when two diagonal elements of H become equal. Besides the standard MSW resonance in the $\nu_{eL} \leftrightarrows \nu_{\mu L}$ channel for $V_{\mathrm{CC}} = \Delta m^2 \cos 2\vartheta / 2E$ (see [1–4]), there are two possibilities

(1) There is a resonance in the $\nu_{eL} \leftrightarrows \nu_{\mu R}$ channel for

$$V_e = \frac{\Delta m^2}{2E}\cos 2\vartheta. \tag{6.23}$$

The density at which this resonance occurs is not the same as that of the MSW resonance, because of the neutral-current contribution to $V_e = V_{\mathrm{CC}} + V_{\mathrm{NC}}$. The location of this resonance depends on both N_e and N_n.

(2) There is a resonance in the $\nu_{\mu L} \leftrightarrows \nu_{eR}$ channel for

$$V_\mu = -\frac{\Delta m^2}{2E}\cos 2\vartheta. \tag{6.24}$$

If $\cos 2\vartheta > 0$, this resonance is possible in normal matter, since the sign of $V_\mu = V_{\mathrm{NC}}$ is negative, as one can see from (6.8).

In practice, the effect of these resonances could be the disappearance of active ν_{eL} or $\nu_{\mu L}$ into sterile right-handed states.

Let us consider now the more interesting case of Majorana neutrinos, which presents two fundamental differences with respect to the Dirac case.

(A) If CP is conserved, Majorana neutrinos can have only a transition magnetic moment $\mu_{e\mu} = -\mu_{\mu e} = -\mu^*_{e\mu}$ in the flavor basis.

(B) The right-handed states are not sterile, but they interact as right-handed Dirac antineutrinos.

Assuming CP conservation, the evolution equation of the amplitudes is given by (6.21) with the effective Hamiltonian matrix:

$$\mathrm{H} = \begin{pmatrix} -\dfrac{\Delta m^2}{4E}\cos 2\vartheta + V_e & \dfrac{\Delta m^2}{4E}\sin 2\vartheta & 0 & \mu_{e\mu}B_{\perp}(x) \\ \dfrac{\Delta m^2}{4E}\sin 2\vartheta & \dfrac{\Delta m^2}{4E}\cos 2\vartheta + V_\mu & -\mu_{e\mu}B_{\perp}(x) & 0 \\ 0 & \mu_{e\mu}B_{\perp}(x) & -\dfrac{\Delta m^2}{4E}\cos 2\vartheta - V_e & \dfrac{\Delta m^2}{4E}\sin 2\vartheta \\ -\mu_{e\mu}B_{\perp}(x) & 0 & \dfrac{\Delta m^2}{4E}\sin 2\vartheta & \dfrac{\Delta m^2}{4E}\cos 2\vartheta - V_\mu \end{pmatrix}. \tag{6.25}$$

Again, besides the standard MSW resonance in the $\nu_{eL} \leftrightarrows \nu_{\mu L}$ channel, there are two possible resonances

(1) There is a resonance in the $\nu_{eL} \leftrightarrows \nu_{\mu R}$ channel for

$$V_{\mathrm{CC}} + 2V_{\mathrm{NC}} = \frac{\Delta m^2}{2E} \cos 2\vartheta. \tag{6.26}$$

(2) There is a resonance in the $\nu_{\mu L} \leftrightarrows \nu_{eR}$ channel for

$$V_{\mathrm{CC}} + 2V_{\mathrm{NC}} = -\frac{\Delta m^2}{2E} \cos 2\vartheta. \tag{6.27}$$

The location of both resonances depend on both N_e and N_n. If $\cos 2\vartheta > 0$, only the first resonance can occur in normal matter, where $N_n \simeq N_e/6$. A realization of the second resonance requires a large neutron number density, as that in a neutron star.

The neutrino spin oscillations in a transverse magnetic field with a possible rotation of the field-strength vector in a plane orthogonal to the neutrino-propagation direction (such rotating fields may exist in the convective zone of the Sun) have been considered in [160–163]. The effect of the magnetic-field rotation may substantially shift the resonance point of neutrino oscillations. Neutrino spin oscillations in electromagnetic fields with other different configurations, including a longitudinal magnetic field and the field of an electromagnetic wave, were examined in [164–169].

It is possible to formulate a criterion [163] for finding out if the neutrino spin and spin-flavor precession is significant for given neutrino and background medium properties. The probability of oscillatory transitions between two neutrino states $\nu_{\alpha L} \leftrightarrows \nu_{\beta R}$ can be expressed in terms of the elements of the effective Hamiltonian matrices (6.22) and (6.25) as

$$P_{\nu_{\alpha L} \leftrightarrows \nu_{\beta R}} = \sin^2\vartheta_{\text{eff}} \sin^2 \frac{x\pi}{L_{\text{eff}}}, \tag{6.28}$$

where

$$\sin^2\vartheta_{\text{eff}} = \frac{4\text{H}_{\alpha\beta}^2}{4\text{H}_{\alpha\beta}^2 + (\text{H}_{\beta\beta} - \text{H}_{\alpha\alpha})^2}, \qquad L_{\text{eff}} = \frac{2\pi}{\sqrt{4\text{H}_{\alpha\beta}^2 + (\text{H}_{\beta\beta} - \text{H}_{\alpha\alpha})^2}}. \tag{6.29}$$

The transition probability can be of order unity if the following two conditions hold simultaneously: (1) the amplitude of the transition probability must be sizable (at least $\sin^2\vartheta_{\text{eff}} \gtrsim 1/2$); (2) the neutrino path length in a medium with a magnetic field should be longer than half the effective length of oscillations L_{eff}. In accordance with this criterion, it is possible to introduce the critical strength of a magnetic field B_{cr} which determines the region of field values $B_\perp > B_{\text{cr}}$ at which the probability amplitude is not small ($\sin^2\vartheta_{\text{eff}} > 1/2$):

$$B_{\text{cr}} = \frac{1}{2\tilde{\mu}} \sqrt{(\text{H}_{\beta\beta} - \text{H}_{\alpha\alpha})^2}, \tag{6.30}$$

where $\tilde{\mu}$ is μ_{ee}, $\mu_{\mu\mu}$, $\mu_{e\mu}$, or $\mu_{\mu e}$ depending on the type of neutrino transition process in question.

Consider, for instance, the case of $\nu_{eL} \leftrightarrows \nu_{\mu R}$ transitions of Majorana neutrinos. From (6.25) and (6.30), it follows [163] that

$$B_{\text{cr}} = \left| \frac{1}{2\tilde{\mu}} \left(\frac{\Delta m^2 \cos 2\vartheta}{2E} - \sqrt{2} G_{\text{F}} N_{\text{eff}} \right) \right|, \tag{6.31}$$

where $N_{\text{eff}} = N_e - N_n$. For getting numerical estimates of B_{cr}, it is convenient to rewrite (6.31) in the following form:

$$B_{\text{cr}} \approx 43\, \frac{\mu_B}{\tilde{\mu}} \left| \cos 2\vartheta \left(\frac{\Delta m^2}{\text{eV}^2} \right) \left(\frac{\text{MeV}}{E} \right) - 2.5 \times 10^{-31} \left(\frac{N_{\text{eff}}}{\text{cm}^{-3}} \right) \right| \text{Gauss}. \tag{6.32}$$

An interesting feature of the evolution equation (6.21) in the case of Majorana neutrinos is that the interplay of spin precession and flavor oscillations can generate $\nu_{eL} \to \nu_{eR}$ transitions [170]. Since ν_{eR} interacts as right-handed Dirac antineutrinos, it is often denoted by $\overline{\nu}_{eR}$, or only $\overline{\nu}_e$, and called "electron antineutrino." This state can be detected through the inverse β-decay reaction:

$$\overline{\nu}_e + p \longrightarrow n + e^+, \tag{6.33}$$

having a threshold $E_{\text{th}} = 1.8\,\text{MeV}$.

The possibility of $\nu_e \to \overline{\nu}_e$ transitions generated by spin-flavor precession of Majorana neutrinos is particularly interesting for solar neutrinos, which experience matter effects in the interior of the Sun in the presence of the solar magnetic field (see [10, 171]). Taking into account the dominant $\nu_e \to \nu_a$ transitions due to neutrino oscillations (see [1–4]), with $\nu_a = \cos\vartheta_{23}\nu_\mu - \sin\vartheta_{23}\nu_\tau$ and $\sin^2 2\vartheta_{23} > 0.95$ (90% C.L.) [66], the probability of solar $\nu_e \to \overline{\nu}_e$ transitions is given by [172]

$$P_{\nu_e\to\overline{\nu}_e} \simeq 1.8\times 10^{-10}\sin^2 2\vartheta_{12}\left(\frac{\mu_{ea}}{10^{-12}\ \mu_B}\ \frac{B_\perp(0.05R_\odot)}{10\ \text{kG}}\right)^2, \tag{6.34}$$

where μ_{ea} is the transition magnetic moment between ν_e and ν_a, $\sin^2 2\vartheta_{12} = 0.857^{+0.023}_{-0.025}$ [66] and $R_\odot$ is the radius of the Sun.

It is also possible that spin-flavor precession occurs in the convective zone of the Sun, where there can be random turbulent magnetic fields [173–175]. In this case [176],

$$\begin{aligned} P_{\nu_e\to\overline{\nu}_e} \approx 10^{-7}S^2\left(\frac{\mu_{ea}}{10^{-12}\ \mu_B}\right)^2\left(\frac{B}{20\ \text{kG}}\right)^2\left(\frac{3\times 10^4\ \text{km}}{L_{\max}}\right)^{p-1} \\ \times\left(\frac{8\times 10^{-5}\ \text{eV}^2}{\Delta m^2_{21}}\right)^p\left(\frac{E}{10\,\text{MeV}}\right)^p\left(\frac{\cos^2\vartheta_{12}}{0.7}\right)^p, \end{aligned} \tag{6.35}$$

where S is a factor of order unity depending on the spatial configuration of the magnetic field, B is the average strength of the magnetic field at the spatial scale $L_{\max}$, which is the largest scale of the turbulence, p is the power of the turbulence scaling, $\Delta m^2_{21} = 7.50^{+0.19}_{-0.20}\times 10^{-5}\,\text{eV}^2$ [66], and E is the neutrino energy. A possible value of p is $5/3$ [173–175], corresponding to Kolmogorov turbulence. Conservative values for the other parameters are $B = 20$ kG and $L_{\max} = 3\times 10^4$ km.

In 2002, the Super-Kamiokande Collaboration established for the flux of solar $\overline{\nu}_e$'s a 90% C.L. an upper limit of 0.8% of the Standard Solar Model (SSM) neutrino flux in the range of energy from 8 to 20 MeV [177] by taking as a reference the BP00 SSM prediction $\phi^{\text{BP00}}_{^8\text{B}} = 5.05\times 10^6\ \text{cm}^{-2}\,\text{s}^{-1}$ for the solar ^{8}B flux [178] and assuming an undistorted ^{8}B spectrum for the $\overline{\nu}_e$'s. This limit was improved in 2003 by the KamLAND Collaboration [179] to 2.8×10^{-4} of the BP00 SSM prediction at 90% C.L. by measuring $\phi_{\overline{\nu}_e} < 370\,\text{cm}^{-2}\,\text{s}^{-1}$ (90% C.L.) in the energy range 8.3–14.8 MeV, which corresponds to $\phi_{\overline{\nu}_e} < 1250\,\text{cm}^{-2}\,\text{s}^{-1}$ (90% C.L.) in the entire ^{8}B energy range assuming an undistorted spectrum.

Recently, the Borexino collaboration established the best limit on the probability of solar $\nu_e \to \overline{\nu}_e$ transitions [180]:

$$P_{\nu_e\to\overline{\nu}_e} < 1.3\times 10^{-4}\quad (90\%\ \text{C.L.}), \tag{6.36}$$

by taking as a reference $\phi^{\text{SSM}}_{^8\text{B}} = 5.88\times 10^6\,\text{cm}^{-2}\,\text{s}^{-1}$ [181] and assuming an undistorted ^{8}B spectrum for the $\overline{\nu}_e$'s. They measured $\phi_{\overline{\nu}_e} < 320\,\text{cm}^{-2}\,\text{s}^{-1}$ (90% C.L.) for $E_{\overline{\nu}_e} > 7.3$ MeV, which corresponds to $\phi_{\overline{\nu}_e} < 760\,\text{cm}^{-2}\,\text{s}^{-1}$ (90% C.L.) in the entire ^{8}B energy range assuming an undistorted spectrum.

The implications of the limits on the flux of solar $\overline{\nu}_e$'s on Earth for the spin-flavor precession of solar neutrinos have been studied in several papers [172–175, 182–185], taking into account the dominant $\nu_e \rightarrow \nu_\mu, \nu_\tau$ transitions due to neutrino oscillations (see [1–4]). Using (6.34) and (6.36), we obtain

$$\mu_{ea} \lesssim 1.3 \times 10^{-12} \, \frac{7 \text{ MG}}{B_\perp(0.05R_\odot)} \, \mu_\text{B}, \tag{6.37}$$

with $600 \text{ G} \lesssim B_\perp(0.05R_\odot) \lesssim 7 \text{ MG}$ [180]. In the case of spin-flavor precession in the convective zone of the Sun with random turbulent magnetic fields, (6.35) and (6.36) give, assuming $p = 5/3$,

$$\mu_{ea} \lesssim 4 \times 10^{-11} \, \text{S}^{-1} \, \frac{20 \text{ kG}}{B} \left(\frac{L_\text{max}}{3 \times 10^4 \text{ km}} \right)^{1/3} \mu_\text{B}. \tag{6.38}$$

The spin-flavor mechanism was also considered [186] in order to describe time variations of solar-neutrino fluxes in Gallium experiments. The effect of a nonzero neutrino magnetic moment is also of interest in connection with the analysis of helioseismological observations [187].

The idea that the neutrino magnetic moment may solve the supernova problem, that is, that the neutrino spin-flip transitions in a magnetic field provide an efficient mechanism of energy transfer from a protoneutron star, was first discussed in [188] and then investigated in some detail in [189–191]. The possibility of a loss of up to half of the active left-handed neutrinos because of their transition to sterile right-handed neutrinos in strong magnetic fields at the boundary of the neutron star (the so-called boundary effect) was considered in [163].

7. Summary and Perspectives

In this review, we have considered the electromagnetic properties of neutrinos with focus on the most important issues related to the problem. The main results discussed in the paper can be summed up as follows.

In the most general case, the neutrino electromagnetic vertex function is defined in terms of four form factors: the charge, dipole magnetic, and electric and anapole form factors. This decomposition is consistent with Lorentz and electromagnetic gauge invariance. The four form factors at zero momentum transfer $q^2 = 0$ are, respectively, the neutrino charge, magnetic moment, electric moment, and anapole moment. These quantities contribute to elements of the scattering matrix and describe neutrino interactions with real photons.

An important characteristic of neutrino electromagnetic properties is that they are different for Dirac and Majorana neutrinos. In particular, Majorana neutrinos cannot have diagonal magnetic or electric moments. Thus, studies of neutrino electromagnetic interactions can be used as a procedure to distinguish whether a neutrino is a Dirac or Majorana particle.

Moreover, CP invariance in the lepton sector puts additional constraints on the neutrino form factors and can be tested with experimental probes of neutrino electromagnetic interactions.

Up to now, no effect of neutrino electromagnetic properties has been found in terrestrial laboratory experiments and in the analyses of astrophysical and cosmological data. However, massive neutrinos have nontrivial electromagnetic properties in a wide set of theoretical frameworks, including the simplest extension of the Standard Model with inclusion of singlet right-handed neutrinos. Therefore, the search for nonvanishing neutrino electromagnetic properties is of great interest for experimentalist and theorists.

The neutrino dipole magnetic (and also electric) moment is theoretically the most well studied and understood among the neutrino electromagnetic moments. In models which extend the Standard Model with the addition of singlet right-handed neutrinos, a Dirac neutrino has a nonzero magnetic moment proportional to the neutrino mass, which yields a very small value for the magnetic moment, less than about $3 \times 10^{-19}\,\mu_B$ for a neutrino mass smaller than 1 eV (3.5). Extra terms contributing to the magnetic moment of the neutrino that are not proportional to the neutrino mass may exist, for example, in the framework of left-right symmetric models. In this type of models, as well as in other generalizations of the Standard Model, as, for instance, in supersymmetric and extradimension models, values of the neutrino magnetic moment much larger than $10^{-19}\ \mu_B$ can be obtained.

The most severe terrestrial experimental upper bound for the effective electron antineutrino magnetic moment, $\mu_{\bar{\nu}_e} \le 2.9 \times 10^{-11}\,\mu_B$ at 90% C.L. (3.31), has been recently obtained in the direct antineutrino-electron scattering experiment performed by the GEMMA collaboration [58]. This value is still about an order of magnitude weaker than the upper bound $\mu_\nu \lesssim 3 \times 10^{-12}\,\mu_B$ (5.8) obtained from astrophysics (considering the cooling of red giant stars) [153].

There is a gap of some orders of magnitude between the present experimental limits $\sim 10^{-11} \div 10^{-12}\,\mu_B$ on neutrino magnetic moments and the predictions of different extensions of the Standard Model which hint at a range $\sim 10^{-14} \div 10^{-15}\,\mu_B$ [28, 29, 80] (see Section (3.5)). The terrestrial experimental constraints have been improved by only one order of magnitude during a period of about twenty years. Further improvements are very important, but unfortunately at the moment there is no new idea which could lead to fast improvements in the near future. On the other hand, astrophysical studies could allow significant improvements of the sensitivity to nontrivial neutrino electromagnetic properties and maybe find a positive indication in their favor. In particular, neutrino flows in extreme astrophysical environments with very strong magnetic fields are sensitive to small values of the neutrino electromagnetic moments. An example is the modelling of neutrinos propagation during core-collapse supernovae (see [192] and references therein) where very strong magnetic fields are believed to exist and in which the influence of neutrino electromagnetic properties has not yet been taken into account.

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